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PREFACE.

EDUCATION is progressive. The development of the popular mind is becoming the transcendent question of the day. Improvements are being made in every department, dull routine is giving way to intellectual activity, instruction is becoming a science, and teaching a profession.

This advance in education has been nownere more noticeable than in the improvements of text-books upon Arithmetic. A few years ago an unpretending little work, Colburn's Intellectual Arithmetic, was presented to the public. That little work touched Arithmetic as with the wand of an enchantress, and transformed it from a dry collection of mechanical processes to a thing of interest and beauty. It laid the foundation of that system of Mental Arithmetic which has infused a new spirit into the science of numbers, and has done more than any other influence to vitalize the methods of common school instruction in this country.

In presenting a new work upon the subject, I desire to acknowledge my obligations to this and other works which have followed it. Bringing to the task the reflection and experience of many years of educational labor, I hope to be able to present a text-book upon Arithmetic which will take an honorable position among the many valuable works upon the subject which are doing so much for the educational interests of the country. Some of the general and special features of this work will be briefly noticed.

METHOD OF TREATMENT.—The method of treatment is both Inductive and Deductive, embracing Analysis and Synthesis. In some cases both of these methods are employed in the development of the same subject; in other cases they are combined in the same solution or explanation, and such combination is characteristic of the entire work. I have endeavored to meet the wants of both teacher and pupil, by preparing a work convenient for instruction, adapted to the natural and logical development of the mind of the pupil in the study of numbers, and containing such applications as will prepare students for the business relations of life.

ARRANGEMENT.—The arrangement of the work is believed to be strictly *logical* and at the same time, *practical*, being adapted to the natural mental growth and development of the pupil. The mottoes have been,—*from the easy to the difficult, from the simple to the complex, from the known to the unknown.* Care has been taken to present the simpler and

more practical subjects first, and not to anticipate any principles or processes before the pupil is prepared for them. Thus, I have placed Compound Numbers after Fractions, Percentage before Ratio and Proportion, Equation of Payments after Proportion, and other arrangements have been determined by the same principle.

THE REASONING.—All reasoning is *comparison*. A comparison requires a standard, and this standard is the *fixed*, the *axiomatic*, the *known*. The law of correct reasoning, therefore, is to compare the *complex* with the *simple*, the *theoretic* with the *axiomatic*, the *unknown* with the *known*. This law is kept prominently before the mind in the development of this work, and upon it are based its definitions, solutions and explanations, etc. As an illustration, notice the definitions of Ratio, Proportion, etc., the method of stating a proportion, etc.

SOLUTIONS.—The solutions and demonstrations are so simple and clear, that they may be understood by very young pupils, yet they are expressed in language concise and logically accurate, and *in the form which the pupil should be required to use at recitation*. A solution may be too concise to be readily understood, and it may also be too prolix, the idea being smothered or concealed in a multiplicity of words. Both of these errors I have endeavored to avoid, remembering that *the highest science is the greatest simplicity*.

RULES.—The rules or methods of operation are expressed in brief and simple language, and are given as the results of solutions and explanations. I have endeavored to lead the pupil to see the reason for the different processes, thus enabling him to derive his own method of operation based upon such reasoning. The object has been to develop mind as well as the power of computation—to make thinkers rather than arithmetical machines.

APPLICATIONS.—One of the most prominent features of the work is its *practical character*. The applications of the science are not the thought of the scholar as what business may be, but represent the *actual business of the day*. Many of the problems and processes are derived from *actual business transactions*. Our *Bills* and *Accounts* came out of the stores; our *Taxes*, *Banking*, *Exchange*, etc., have been submitted to and endorsed by those connected with the business; several of the problems on *Duties* are out of the *Custom House; Insurance* has been examined by experts in the business; the subject of *Building Associations*, for the first time introduced into an arithmetic, was partly prepared by one practically familiar with the subject; etc.

UNION OF MENTAL AND WRITTEN.—Another leading feature of the work is the union of mental and written arithmetic in one book. Many who recognize the importance of Mental Arithmetic think that it takes too much time for the pupil to study two separate books—one on Men-

tal and the other on Written Arithmetic—and hold that these two subjects should be embraced in *one book*. To meet this demand I have made a complete and harmonious combination of the two subjects, introducing many of those forms of analysis that have given such popularity to my Mental Arithmetic. It is this combination that gives the work its name, *The Union Arithmetic;* and this union will be found to be not a mere nominal thing, but a reality. In the study of the work the pupil can obtain quite a thorough course in *arithmetical analysis* while he is becoming familiar with the art of computation and the application of the art to business. These mental exercises are so arranged and printed that any teacher who prefers to omit them can do so without any inconvenience to either the pupil or the teacher.

SPECIAL FEATURES.—There are several special features peculiar to this work, to which we desire to call attention.

1st. Many new definitions, as of Fraction, Least Common Multiple, Percentage, Ratio, etc.

2d. New and concise method of explaining Greatest Common Divisor, and a method of Least Common Multiple not usually given.

3d. The two distinct methods of the development of Fractions, the *relation* of fractions, the method of stating a problem in Simple Proportion and reason for it, and the development of Compound Proportion.

4th. The Analytic and Synthetic methods of developing Involution and Evolution, the greater attention to Involution as a preparation to Evolution, a new method of cube root, etc.

5. Great number and variety of problems, especially after the Fundamental Rules, Fractions, etc., and at the close of the book. Other features also important, will present themselves upon a careful examination.

It should be stated that this work was first published in 1863, and that some of the definitions and processes which were then new have since been introduced into other works. The present edition is thoroughly revised, and brought up to the very latest methods of business calculations.

Thanking my friends for the cordial reception given to my previous labors, I send forth this new volume, with the earnest desire that it may meet their approbation, and aid in the development and diffusion of a deeper interest in the beautiful science of numbers—a science which practically lies at the foundation of all science and all thought, and one which is doing so much to promote the cause of popular education.

EDWARD BROOKS.

STATE NORMAL SCHOOL,
May 10, 1877.

TO TEACHERS.

Teachers will notice that the author's *Graded Series* of arithmetics consists of three parts. Having completed the Primary Arithmetic, or Part I., which is an introductory course in the science for beginners, the pupil is prepared to take up the present volume, Part II. Part II. begins at the beginning of arithmetic and extends as far as Percentage. It embraces Numeration and Notation, Fundamental and Derivative Operations, Fractions, Decimals and their applications, Denominate Numbers, and Practical Measurements. After completing Part II., the pupil will be prepared to take up Part III., which begins at Percentage and finishes the course in arithmetic. These two parts are also bound in one volume for those who prefer the book in that form.

THE

NORMAL

UNION ARITHMETIC

SECTION I.

ARITHMETICAL LANGUAGE.

1. **Arithmetic** is the science of numbers and the art of computing with them.

2. A **Unit** is a single thing or *one*. A thing is a *concrete unit*; *one* is an *abstract unit*.

3. A **Number** is a unit or a collection of units. Numbers are *concrete* and *abstract*.

4. A **Concrete Number** is one in which the kind of unit is named; as, two *yards*, five *books*.

5. An **Abstract Number** is one in which the kind of unit is not named; as, *two*, *four*, etc.

6. **Similar Numbers** are those in which the units are alike; as, two *boys* and four *boys*.

7. **Dissimilar Numbers** are those in which the units are unlike; as, two *boys* and four *books*.

8. A **Problem** is a question requiring some unknown result from that which is known.

9. A **Solution** of a problem is a process of obtaining the required result.

10. A **Rule** is a statement of the method of solving a problem.

11. Mental Arithmetic treats of performing arithmetical operations without the aid of written characters.

12. Written Arithmetic treats of performing arithmetical operations with written characters.

13. Arithmetical Language is the method of expressing numbers.

14. Arithmetical Language is of two kinds, *Oral* and *Written*. The former is called *Numeration* and the latter is called *Notation*.

NOTE.—A number is really the *how many* of the collection instead of the *collection*; but the definition given, which is a modification of Euclid's, is simpler and sufficiently accurate.

NUMERATION.

15. Numeration is the method of naming numbers, and of reading them when expressed by characters. It is the *oral expression* of numbers.

16. Since it would require too many words to give each number a separate name, numbers are named according to the following simple principle:

Principle.—*We name a few of the first numbers, and then form groups or collections, name these groups, and use the names of the first numbers to number these groups.*

17. A single thing is named *one*: one and one more are named *two*; two and one more, *three*; three and one more, *four*; and thus we obtain the simple names,

One, two, three, four, five, six, seven, eight, nine, ten.

18. Now, regarding the collection *ten* as a single thing, we might count *one and ten, two and ten, three and ten*, etc., as far as *ten and ten*, which we would call *two tens*. By this principle were obtained the following numbers:

Eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, twenty.

19. Proceeding in the same way, we would have *two tens and one, two tens and two, two tens and three*, etc. By this principle were obtained the following numbers:

Twenty-one, twenty-two, twenty-three, twenty-four, twenty-five, twenty-six, twenty-seven, twenty-eight, twenty-nine.

20. Continuing in the same manner, we would have *three-tens, four-tens, five-tens*, etc. By this principle were derived the following ordinary names:

Thirty, forty, fifty, sixty, seventy, eighty, ninety.

21. A group of *ten tens* is called a *hundred;* a group of *ten hundreds*, a *thousand;* the next group receiving a new name consists of a *thousand thousands*, called a *million;* the next group of a *thousand millions*, called a *billion*, etc.

22. After a *thousand*, the two intermediate groups between those having a distinct name, are numbered by *tens* and *hundreds*, as *ten thousand* and *hundred thousand.*

NOTES.—1. The above shows the *principle* by which the numbers were named. The names, however, were not derived from the particular expressions given, but originated in the Saxon language.

2. *Eleven* is from the Saxon *endlefen*, or Gothic *ainlif* (*ain*, *one*, and *lif*, *ten*); *twelve* is from the Saxon *twelif*, or Gothic *tvalif* (*tva*, *two*, and *lif*, *ten*). Some have supposed that *eleven* meant *one left after ten*, and *twelve*, *two left after ten.*

3. *Twenty* is from the Saxon *twentig* (*twegen*, *two*, and *tig*, a *ten*); *thirty* is from the Saxon *thritig* (*thri*, *three*, and *tig*, a *ten*), etc.

4. *Hundred* is a primitive word; *thousand* is from the Saxon *thusend*, or Gothic *thusundi*, (*thus*, *ten*, and *hund*, *hundred*); *million*, *billion*, etc., are from the Latin.

NOTATION.

23. Notation is the method of writing numbers. Numbers may be written in three ways:

1st. By *words*, or common language.

2d. By *figures*, called the *Arabic Method.*

3d. By *letters*, called the *Roman Method.*

NOTE.—The method by words is that of ordinary written language and needs no explanation.

ARABIC NOTATION.

24. The **Arabic System** of Notation is the method of expressing numbers by characters called *figures.*

25. In this system numbers are expressed according to the following principle:

Principle.—*We employ characters to represent the first nine numbers, and then use these characters to number the groups, the group numbered being indicated by the position of the character.*

26. Figures.—Figures are characters used in expressing numbers. There are ten figures used, as follows:

FIGURES..	1,	2,	3,	4,	5,	6,	7,	8,	9,	0.
NAMES AND VALUES.	one,	two,	three,	four,	five,	six,	seven,	eight,	nine,	naught, cipher or zero.

27. By the combination of these figures all numbers may be expressed; hence they are appropriately called the *alphabet of arithmetic.*

28. Combination.—These figures are combined according to the following principle:

1. *A figure standing alone, or in the first place at the right of other figures, expresses* UNITS *or* ONES.

2. *A figure standing in the second place, counting from the right, expresses* TENS; *in the third place,* HUNDREDS; *in the fourth place,* THOUSANDS, etc.; thus,

10 is 1 ten, or ten.	100 is 1 hundred.
20 " 2 tens, or twenty.	200 " 2 hundred.
30 " 3 tens, or thirty.	300 " 3 hundred.
40 " 4 tens, or forty.	400 " 4 hundred.
50 " 5 tens, or fifty.	1000 " 1 thousand.
60 " 6 tens, or sixty.	2000 " 2 thousand.
90 " 9 tens, or ninety.	4000 " 4 thousand.

29. The name of each of the first twenty-one places is represented by the following

NUMERATION TABLE.

	Hundred-quintillions.	Ten-quintillions.	Quintillions.	Hundred-quadrillions.	Ten-quadrillions.	Quadrillions.	Hundred-trillions.	Ten-trillions.	Trillions.	Hundred-billions.	Ten-billions.	Billions.	Hundred-millions.	Ten-millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.
	8	8	8,	8	8	8,	8	8	8	8	8	8,	8	8	8,	8	8	8,	8	8	8,
PLACES.	21st,	20th.	19th.	18th,	17th,	16th.	15th,	14th,	13th,	12th,	11th,	10th,	9th,	8th,	7th,	6th,	5th,	4th,	3d,	2d,	1st,
PERIODS.	7th.			6th.			5th.			4th.			3d.			2d.			1st		

30. Periods.—For convenience in writing and reading numbers, the figures are arranged in *periods* of three places each, as shown in the table. The first three places constitute the *first*, or *units period;* the second three places constitute the *second*, or *thousands period*, etc.

1. Required the names of the following places:

First; third; second; sixth; fourth; eighth; tenth; ninth; twelfth; fifth; seventh; eleventh; thirteenth; seventeenth; fourteenth; sixteenth; eighteenth; fifteenth; nineteenth; twenty-first; twentieth.

2. Required the places of the following:

Tens; hundreds; thousands; millions; ten-thousands; hundred thousands; ten-millions; billions; hundred-millions; hundred-billions; units; ten-billions; trillions; quadrillions; hundred-quintillions; ten-trillions; ten-quintillions; hundred-quadrillions; quintillions; hundred-trillions; ten-quadrillions.

3. Required the names of the following periods:

1. First.	3. Second.	5. Fourth.
2. Third.	4. Fifth.	6. Seventh.

4. Required the period and place of the following:

Thousands; millions; ten-thousands; hundred-millions; billions; hundred-trillions; trillions; ten-trillions; quadrillions; ten-quadrillions; hundred-trillions; quintillions; hundred-quintillions; hundred-thousands; ten-millions.

31. The combination of figures to express a number forms a *numerical expression.* Thus, 25 is a numerical expression which denotes the same as the common word *twenty-five.*

32. The different figures of a numerical expression are called *terms.* *Terms* are also used to indicate the *numbers* represented by the figures.

NOTE.—The use of the word *term*, to indicate both the figures and the numbers represented by them, enables us to avoid the error of using the word *figure* for the word *number.*

EXERCISES IN NUMERATION.

33. The pupils are now prepared to learn to *read* numbers when expressed by *figures.* From the preceding explanations we have the following *rule for numeration:*

Rule.—I. *Begin at the right hand, and separate the numerical expression into periods of three figures each.*

II. *Then begin at the left hand and read each period in succession, giving the name of each period except the last.*

NOTE.—The name of the last period is usually omitted, because it is understood.

1. What number is expressed by 6325478?

SOLUTION.—Separating the numerical expression into periods of three figures each, beginning at the right hand, we have 6,325,478. The third period is 6 *millions*, the second period is 325 *thousands*, and the first is 478 *units*, hence the number is 6 *millions*, 325 *thousands*, 478.

OPERATION.
6,325,478

Read the following numerical expressions:

2.	3876	10.	468217	18.	80305072
3.	2185	11.	654879	19.	65073058
4.	3072	12.	803006	20.	484378513
5.	5678	13.	1234567	21.	123456789
6.	12630	14.	3078560	22.	854327031
7.	70851	15.	8507032	23.	80735468579
8.	32468	16.	54372568	24.	20650708462067
9.	507035	17.	87072135	25.	798653013285678521

NOTE.—After pupils are familiar with reading by dividing into periods, the division may be omitted or performed mentally.

EXERCISES IN NOTATION.

34. Having learned to *read* numerical expressions, we are now prepared to *write* them. From the principles which have been explained, we derive the following rule:

Rule.—I. *Begin at the left and write the hundreds, tens, and units of each period in their proper order.*

II. *When there are vacant places, fill them with ciphers*

1. Express in figures the number *four thousand three hundred and four.*

SOLUTION.—We write the 4 thousands in the 4th place, 3 hundreds in the 3d place, a cipher in the 2d place, there being no tens, and 4 units in the 1st place, and we have 4304.

OPERATION.
4,304

Express the following numbers in figures:

2. One hundred and six.
3. One hundred and ten.
4. Two hundred and forty.
5. Two hundred and sixty-five.
6. Two hundred and nine.
7. Three hundred and twelve
8. Three hundred and fifty seven.
9. Four hundred and twenty eight.
10. Seven hundred and eighty four.

11. Nine hundred and thirty-seven

12. Eight hundred and ninety-nine.

13. Four thousand and seven.

14. Five thousand two hundred and thirty-six.

15. Six thousand and eighty-five.

16. Twenty-three thousand six hundred and forty-seven.

17. One hundred and forty-five thousand seven hundred and six.

18. Three hundred and eight thousand three hundred and eight.

19. Six hundred and four thousand three hundred and sixty-eight.

20. Eight hundred and seventy-four thousand one hundred and twenty.

21. Seven hundred and seven thousand seven hundred and seven.

22. One million.

23. Two million and six.

24. Three million and twelve.

25. Forty-five million and twenty-four.

26. Six million forty-seven thousand.

27. Eight million two thousand and sixty-seven.

28. Five million two hundred and ninety-six thousand.

29. Seventy million, one thousand and forty-five.

30. One million, two hundred and thirty thousand, four hundred and fifty-six.

31. Four million, three hundred and seven thousand, four hundred and nine.

32. Fifteen million, four hundred and eight thousand, and eighty-four.

33. Twenty-eight million, five hundred and ninety-four thousand, seven hundred and nine.

34. Forty-seven million, thirty-eight thousand, two hundred and eight.

35. Two hundred million, forty-nine hundred and twenty-eight.

36. Six billion, seven hundred and five million, thirty-five thousand and six.

37. Forty-nine trillion, fifty-eight thousand, seven hundred and ninety-eight.

35. Orders.—Since we may have 2 *tens*, 3 *tens*, etc., 2 *hundreds*, 3 *hundreds*, etc., the same as 2 *apples*, 3 *apples*, 2 *books*, 3 *books*, etc., these different groups may be regarded as *units* of different *orders;* thus,

UNITS	are called	Units of the 1*st order.*
TENS	" "	Units of the 2*d order.*
HUNDREDS	" "	Units of the 3*d order.*
THOUSANDS	" "	Units of the 4*th order.*
TEN-THOUSANDS	" "	Units of the 5*th order.*
etc.,		etc.

...ten units of a lower order
...ten of the next higher order; the system of notation
...the Decimal System, from the Latin.

WRITTEN EXERCISES.

...thousands? In 5 thousands? In
...millions? In 3 millions?

2. What is the result of chang-
...one place to the left
...one place to the left? Three places?

3. What is the result of chang-
...one place to the right? Three places?

4. What is the result of plac-
...ten next to the right of one
...ten? Two naughts?

5. What place is occupied by
...hundreds figure? By
...millions? By

THE DECIMAL SCALE.

38. Since figures express different orders of units in different places, they are said to have two kinds of values; a *simple value* and a *local value.*

39. The **Simple Value** of a figure is its value when it stands alone or in units place. The *Local Value* of a figure is its value when in some other place than units.

40. The value of a figure is *increased tenfold* for every place it is moved from *right to left;* and is *decreased tenfold* for every place it is moved from *left to right.* This law is called the *scale* of the system.

41. Since the value of terms decreases from left to right at a tenfold rate, if we fix the place of units by a point (.), we may extend the decimal scale to the right of units.

42. The first place on the right of the point will be one-tenth of units or *tenths*, the second place one-tenth of tenths, or *hundredths*, the third place, *thousandths*, etc.

43. Such terms are called *decimals*, and the point is called the *decimal point.* The expression 48.37 is read 48 units, 3 tenths, and 7 hundredths, or 48 and 37 hundredths.

44. The **Money** of the United States is expressed by the decimal system. The *dollar* is the *unit*, and is indicated by the symbol $. The first place at the right of the decimal point is called *dimes;* the second place, *cents*, etc.

45. **Dimes** and **Cents** are usually read as a *number of cents.* Thus, $4.65 is read 4 dollars and 65 cents; and 72.48 is read 72 dollars and 48 cents.

The Decimal system of numeration had its origin in the practice, common to all nations, of counting by groups of tens.

EXAMPLES FOR PRACTICE.

Read the following:

12.5	5.	46.375	9.	$125.45
36.25	6.	89.625	10.	$437.05
47.75	7.	$16.25	11.	$548.475
86.50	8.	$85.35	12.	$768.605

Write the following:

1. Fourteen and five tenths.
2. Eighty-four and twenty-five hundredths.
3. Two hundred and three, and sixty-seven hundredths.
4. Seven hundred and ninety-six, and eight hundred seventy-five thousandths.
5. Sixty dollars and seven cents.
6. Eighty-seven dollars and twenty-five cents.
7. Four hundred and fifty dollars, and fifty cents.
8. Eight hundred and sixty-four dollars, thirty-seven cents and five mills.

NOTE.—It is recommended that only advanced or finishing classes study the English Method of Numeration.

ENGLISH METHOD OF NUMERATION.

46. The method of numeration by dividing numbers into periods of three figures each, is called the *French Method.*

47. The **English Method** uses periods of six figures each, calling the first period *units*, the second *millions*, the third *billions*, the fourth *trillions*, etc.

48. The places in each period are *units, tens, hundreds, thousands, tens of thousands, hundreds of thousands.* The method is represented in the following table:

Hund. of Thou. of Trillions.	Tens of Thou. of Trillions.	Thousands of Trillions.	Hundreds of Trillions.	Tens of Trillions.	Trillions.	Hund. of Thou. of Billions.	Tens of Thou. of Billions.	Thousands of Billions.	Hundreds of Billions.	Tens of Billions.	Billions.	Hund. of Thou. of Millions.	Tens of Thou. of Millions.	Thousands of Millions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
6	6	6	6	6	6,	6	6	6	6	6	6,	6	6	6	6	6	6,	6	6	6	6	6	6,
4th or Trillions Period.						3d or Billions Period.						2d or Millions Period.						1st or Units Period					

NOTE.—The French Method is used throughout the United States, France, etc., and being much more convenient, is very likely to supersede the other method in England.

EXAMPLES FOR PRACTICE.

1. Write by the English method, one million; one billion; one trillion; one quadrillion.

ROMAN NOTATION.

49. The **Roman Method** of Notation employs seven letters of the Roman alphabet. Thus, I represents *one;* V, *five;* X, *ten;* L, *fifty;* C, *one hundred;* D, *five hundred;* M, *one thousand.*

50. To express other numbers these characters are combined according to the following principles:

1. *Every time a letter is repeated its value is repeated.*
2. *When a letter is placed before one of greater value, the* DIFFERENCE *of their value is the number represented.*
3. *When a letter is placed after one of a greater value, the* SUM *of their values is the number represented*
4. *A dash placed over an expression increases its value a thousand fold.* Thus, $\overline{\text{VII}}$ denotes seven thousand.

51. These principles are exhibited in the following table, which the pupil will examine carefully:

ROMAN TABLE.

I . . .	One.	XXX .	Thirty.
II . . .	Two.	XL .	Forty.
III . . .	Three.	L .	Fifty.
IV . . .	Four.	LX .	Sixty.
V . . .	Five.	LXX .	Seventy.
VI . . .	Six.	XC .	Ninety.
VII . . .	Seven.	C .	One hundred.
VIII . .	Eight.	CC .	Two hundred.
IX . . .	Nine.	D .	Five hundred.
X . . .	Ten.	DC .	Six hundred.
XI . . .	Eleven.	DCCCC	Nine hundred.
XIV . .	Fourteen.	M .	One thousand.
XV . .	Fifteen.	MM .	Two thousand
XIX . .	Nineteen.	MCLX	One thousand one hundred
XX . .	Twenty.	MDCCCLXXVI, 1876	[and sixty

52. The **Roman Method** is named from the Romans, who invented and used it. It is now only employed to denote the chapters and sections of books, pages of preface and introduction, and in other places for prominence and distinction.

WRITTEN EXERCISES.

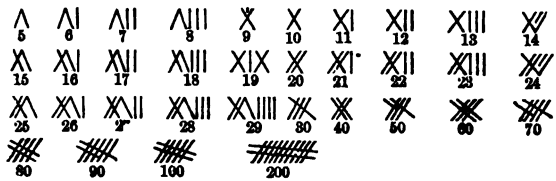
Express the following numbers by the Roman method: 1. Twenty-seven. 2. Seventy-seven. 3. Two hundred and one. 4. Six hundred and fifty-six. 5. One thousand seven hundred and seventy-six. 6. Four thousand seven hundred and fifty-seven. 7. 25007. 8. 206484.

Read the following numbers:

1. LXXVII. 2. MXC. 3. MDCCCLXXVI. 4. MMMCCCXXXIII. 5. $\overline{X}$VDCCXLIV. 6. clxxxviii. 7. xlix. 8. xcix.

LUMBERMEN'S NOTATION.

53. Lumbermen in marking lumber employ a modification of the Roman Method of Notation. The first four characters are like the Roman. The others are as follows:



NOTE.—For a full discussion of Arithmetical Language, see the author's *Philosophy of Arithmetic.*

INTRODUCTION TO ADDITION.

MENTAL EXERCISES.

1. If there are 7 boys in one class and 6 in another class, how many boys in both classes?

SOLUTION.—If there are 7 boys in one class and 6 in another class, in both classes there are 7 boys plus 6 boys, which are 13 boys.

2. In a garden there are 9 apple-trees and 10 pear-trees; how many trees in the garden?

3. Anna bought a bonnet for 15 dollars and a cloak for 24 dollars; how much did both cost her?

4. Mary gave 19 apples to one of her schoolmates and 15 to another; how many did she give to both?

5. If a boy gave 75 cents for a pair of skates and 25 cents for a ball, what did they both cost him?

6. A lady gave 10 cents for needles, 14 cents for thread, and 15 cents for muslin; what did she give for all?

7. A farmer sold some wheat for 15 dollars, some corn for 25 dollars, and some oats for 30 dollars; what did he receive?

8. How many are 6 and 21? 9 and 22? 8 and 31? 10 and 17? 4 and 18? 7 and 27? 9 and 28? 12 and 11? 13 and 11?

9. How many are 6 and 17? 8 and 16? 9 and 18? 8 and 19? 7 and 12? 10 and 21? 12 and 20? 14 and 18? 11 and 21?

10. How many are 3 and 14? 4 and 17? 5 and 15? 7 and 17? 6 and 16? 8 and 18? 9 and 19? 11 and 21? 10 and 20?

11. Count by 2's from 2 to 20; from 20 to 40; from 40 to 60; from 1 to 21; from 21 to 41; from 41 to 61.

12. Count by 3's from 3 to 21; from 21 to 42; from 1 to 22; from 22 to 43; from 2 to 23; from 23 to 44.

13. Count by 4's from 4 to 40; from 40 to 80; from 1 to 25; from 25 to 41; from 2 to 30; from 30 to 50; from 3 to 35; from 35 to 55.

14. Count by 5's from 5 to 60; from 1 to 61; from 2 to 62; from 3 to 63; from 4 to 64.

15. Count by 6's, 7's, 8's, 9's, 10's, 11's, and 12's, as in the previous problems.

The uniting of two or more numbers into one sum is called *Addition*. The sign of addition is +, and is read *plus*.

Find the sum of

3+5+4	6+8+5	8+7+6	10+5+7	13+ 6+ 8
4+3+6	3+9+6	7+9+8	11+6+5	14+ 5+ 9
6+7+5	5+7+8	8+9+7	12+7+8	15+ 8+ 7
5+8+2	4+9+7	9+7+8	12+8+9	16+17+18

SECTION II.

FUNDAMENTAL OPERATIONS.

ADDITION.

54. Addition is the process of finding the *sum* of two or more numbers.

55. The **Sum** of several numbers is a number which contains as many units as the numbers added.

56. The **Sign of Addition** is +, and is read *plus*. It denotes that the numbers between which it is placed are to be added.

57. The **Sign of Equality** is =, and is read *equals*. Thus, 10=4+6 is read 10 *equals* 4 *plus* 6.

NOTES.—1. The *Sign of Addition* consists of two short lines bisecting each other, the one in, and the other perpendicular to, the line of writing.
2. The symbol + was introduced by *Stifelius*, a German mathematician, in a work published in 1544.

PRINCIPLES.

1. *The numbers added must be similar.*
2. *The sum is a number similar to the numbers added.*

CASE I.

58. ***To add when the sum of no column exceeds nine units of that column.***

1. What is the sum of 34, 23, and 32?

SOLUTION.—We write the numbers so that figures of the same order stand in the same column, draw a line beneath, and begin at the right to add. 2 units and 3 units are 5 units, and 4 units are 9 units, which we write under the column of units; 3 tens and 2 tens are 5 tens, and 3 tens are 8 tens, which we write under the column of tens. Hence the sum of 34, 23, and 32 is 8 tens and 9 units, or eighty-nine. Hence the following

OPERATION.

34
23
32
89

Rule.—I. *Write the numbers to be added so that terms of the same order stand in the same column.*

II. *Begin at the right, add each column separately, and write each sum under the column which produced it.*

WRITTEN EXERCISES.

(2)	(3)	(4)	(5)	(6)
526	232	328	437	423
243	456	671	312	536
769				

(7)	(8)	(9)	(10)	(11)
$1.23	$5.42	$12.20	$20.06	$132.25
4.31	1.05	23.25	31.20	245.03
2.03	3.50	30.03	46.52	400.50
7.57				

12. What is the sum of $21.35, $15.02, and $50.61?

Ans. $86.98.

13. What is the sum of $32.16, $21.50, and $24.23?

Ans. $77.89.

14. What is the sum of 2131236, 2314010, and 4520652?

Ans. 8965898.

15. A man gave his son $13.56 and his daughter $46.21; now many dollars did he give to both? *Ans.* $59.77

16. Marie's book contains 3516 words, and Jennie's book 4271 words; how many words in both? *Ans.* 7787.

17. John takes 3250 steps in going to school, and Henry takes 2518 steps; how many do both take? *Ans.* 5768.

18. If Mr. Martin rides 3164 miles in a year and walks 513 miles; how far does he travel in a year? *Ans.* 3677.

19. A house cost $4160, a barn $1702, and a store $2027; required the cost of all. *Ans.* $7889.

20. A book cost $2.25, a kite $1.10 and a ball 62 cents; what was the cost of all? *Ans.* $3.97.

21. A little girl's pulse beats 105216 times in a day, and her mother's pulse beats 93470 times; how many beats do both make in a day? *Ans.* 198686.

22. John bought a Higher Arithmetic for $1.38, a Peterson's Familiar Science for $1.60, and a Loomis's Algebra for $2.00; how much did he pay for all? *Ans.* $4.98.

23. An army consisting of 24,360 men received at one time a reinforcement of 10,220, and at another time of 5400; of how many men did it then consist? *Ans.* 39,980

CASE II.

59. ***To add when the sum of a column exceeds nine units of that column.***

1. What is the sum of 475, 384, and 896?

SOLUTION.—We write the numbers so that terms of the same order stand in the same column, and begin at the right to add. 6 units and 4 units are 10 units, and 5 units are 15 units, which equal 1 *ten* and 5 *units;* we write the 5 units under the column of units, and add the 1 ten to the column of tens: 1 ten and 9 tens are 10 tens, and 8 tens are 18 tens, and 7 tens are 25 tens, which equal 2 *hundreds* and 5 *tens;* we write the 5 tens under the column of tens, and add the 2 hundreds to the column of hundreds: 2 hundreds and 8 hundreds are 10 hundreds, and 3 hundreds are 13 hundreds, and 4 hundreds are 17 hundreds, which equal 1 *thousand* and 7 *hundreds;* we write the 7 hundreds under the column of hundreds, and place the 1 thousand on the left in the place of thousands. Hence, the sum of 475, 384, and 896 is 1755.

OPERATION.

475
384
896
1755

Rule.—I. *Write the numbers to be added so that terms of the same order stand in the same column, and draw a line beneath.*

II. *Begin at the right, add the terms of each column separately, and write the sum underneath, if less than ten.*

III. *When the sum of any column is ten or more than ten, write the units figure only, and add the tens to the next column.*

IV. *Write the entire sum of the last column.*

NOTE.—In adding dollars and cents, dollars must be written under dollars and cents under cents, so that the points may be in a vertical line.

Proof.—Begin at the top and add the columns downward and if the work is correct the two sums will be equal.

SECOND METHOD. Separate the number into two or more parts, add these parts, and then add the sum of these parts; and if the work is correct the two results will be equal.

NOTES.—1. We write figures of the same order in the same column for *convenience* of adding, since only units of the same order can be directly added.

2. We begin at the *right* to add for *convenience*, so that when the sum of any column exceeds 9 we may add the left hand term of such sum to the next column.

WRITTEN EXERCISES.

2. What is the sum of 4326+7428+4675+3857+1231?

Ans. 21517

3. What is the sum of 3213+3562+4572+4207+3208?
Ans. 18762.

4. What is the sum of 2573+3865+2101+5213+1202?
Ans. 14954.

5. What is the sum of 3216+3105+21678+36102?
Ans. 64101.

6. What is the sum of 3120+1356+2013+206+12345?
Ans. 19040.

7 Find the sum of 3123+204+36512+30567+123457.
Ans. 193863.

8. What is the sum of 21650+1456+287657+3608+146075? *Ans.* 460446.

9. Find the sum of 60758+12473+10572+876543+752075+875432. *Ans.* 2587853.

10. What is the sum of 67086+1257321+8752072+516203+2010507? *Ans.* 12603189.

11. What is the sum of 124607+576078+1555+6753209+12460729? *Ans.* 19916178.

12. Find the sum of 623405+328721+6075+2870572+395. *Ans.* 3829168.

13. What is the sum of 351670+8037508+367+12578+246+312? *Ans.* 8402681.

14. Required the sum of 2187587+3057218+207562+17473+312578. *Ans.* 5782418.

15. Find the sum of 875672+3408+13767+870823+138721+85432785. *Ans.* 87335176.

16. What is the sum of 8243+13750+372+82134+28793+876789? *Ans.* 1010081.

17. Required the sum of 3207+28543+2856+28732+87256+387128. *Ans.* 537722.

18. Find the sum of 8723+285768+36124+28765+37124+856078. *Ans.* 1252582.

19. What is the sum of 23456+7891011+121314+151617+1819202? *Ans.* 10006600.

20. Required the sum of 28567+385602+13728+38562+308029+2085703. *Ans.* 2860191.

21. Required the sum of 3142+2813+93104+160532++210356+213165. *Ans.* 683112.

22. Find the sum of 36875+28764+3128+6785+3876+38521+46212. *Ans.* 164161.

23. Find the sum of 41287, 32104, 80010, 678910, 136710, 896343. 257856. *Ans* 2123220

24. Find the sum of 67851, 21856, 38214, 621857, 36724, 128705, 1250641. *Ans.* 2165848.

25. Find the sum of 20875, 318764, 287656, 318567, 208725, 387124, 411403. *Ans.* 1953114.

26. Find the sum of 218765, 865432, 896, 32481, 2879823, 8128765, 1759824. *Ans.* 13885986.

27. Find the sum of 61857, 30876, 14687, 318572, 856, 30876, 12345, 678910, 328764. *Ans.* 1477743.

MENTAL EXERCISES.

1. Rose gave 15 cents to her brother and 8 cents to her sister; how much did she give to both?

SOLUTION.—If Rose gave 15 cents to her brother and 8 cents to her sister, to both she gave 15 cents plus 8 cents, which are 23 cents.

2. What is the cost of a suit of clothes if the material cost $25 and the making cost $10?

3. If a boy has 12 cents and then earns 15 cents, how many cents will he then have?

4. The head of a fish is 6 inches long, the tail 8 inches, and the body 12 inches; what is the length of the fish?

5. How many are 18 and 6? 17 and 8? 21 and 7? 25 and 8? 36 and 8? 42 and 7? 48 and 6? 57 and 7?

6. How many are 8 and 24? 9 and 26? 7 and 35? 6 and 48? 5 and 47? 4 and 48? 3 and 68?

7. How many are 105 and 6? 108 and 7? 115 and 8? 124 and 6? 135 and 5? 140 and 7? 155 and 8?

8. How many are 6 and 135? 5 and 206? 7 and 208? 4 and 212? 7 and 234? 8 and 326? 300 and 400?

9. Howard shot 11 robins, Howell shot 12 robins, and Hoyt shot 13 robins; how many robins did they all shoot?

10. A boy bought a robin for 10 cents, a jay for 20 cents, and a wren for 30 cents; what did they all cost?

11. A lady gave 6 cents for needles, 12 cents for thread, and 25 cents for some muslin; what was the entire cost?

12. What is the sum of 28 and 37?

SOLUTION.—28 and 30 are 58, and 7 are 65.

13. What is the sum of 25 and 38? 27 and 36? 45 and 28? 47 and 65? 67 and 48? 73 and 48?

14. What is the sum of 29 and 46? 28 and 38? 76 and 46? 85 and 91? 83 and 48? 96 and 69? 125 and 43? 173 and 205?

15. Add by 7's from 7 to 98; from 1 to 99; from 2 to 100; from 3 to 101. Add by 8's from 8 to 120; from 2 to 122; from 3 to 123.

16. Add by 9's from 0 to 108; from 1 to 109; from 2 to 110; from 3 to 111. Add by 11's from 0 to 132; from 1 to 133; from 2 to 134.

17. Add by 12's from 0 to 144; from 1 to 145; from 2 to 146; from 3 to 147; by 13's from 13 to 130; from 1 to 131; etc.

18. Add alternately by 4 and 5 till the sum exceeds 100; by 5 and 6; by 6 and 7; by 8 and 9; by 9 and 10; by 11 and 12.

19. A merchant sold some rice for $15, some sugar for $17, and some flour for $22; what was the amount of the sale?

WRITTEN EXERCISES.

1. A farmer raised 857 bushels of wheat, 372 bushels of rye, 720 bushels of oats, and 876 bushels of corn; how many bushels of grain did he raise?

SOLUTION.—If he raised 857 bushels of wheat, 372 bushels of rye, 720 bushels of oats, and 876 bushels of corn, he raised in all the sum of 857 bushels, 372 bushels, 720 bushels, and 876 bushels, which we find by adding, to be 2825 bushels.

OPERATION

857
372
720
876
2825

2. A lady owns a farm worth $8326, a house worth $6575, a barn worth $3818, and $7584 in State bonds; what is she worth? *Ans.* $26303.

3. A grocer paid $897 for flour and $759 for sugar; for what must both be sold that the grocer may gain $125 on the transaction? *Ans.* $1781.

4. In a graded school there are 29 pupils in the first section, 42 in the second, 38 in the third, 45 in the fourth, 51 in the fifth, and 63 in the sixth; how many pupils are there in the whole school? *Ans.* 268 pupils.

5. A merchant "checked" out of bank $2560 one day, $3065 the next, $2780 the next, and then had $5760 in bank: how much had he in bank at first? *Ans.* $14165

6. A dealer bought 450 barrels of wheat flour for $2700, and 286 barrels of rye flour for $1416; how many barrels did he buy and what was the whole cost? *Ans.* $4116.

7. A gentleman dying left his eldest son $7286, the next younger son $9111, and the youngest as much as both the others; what was the fortune? *Ans.* $32794.

8. Bought calico for a dress for $2, a neck-tie for $0.45, a parasol for $3.50, a pair of boots for $4.25, and a pair of gloves for $1.75; what was the whole cost? *Ans.* $11.95.

9. A speculator bought three city lots for $5750 each; for what must he sell them that he may gain $2367 by the investment? *Ans.* $19617.

10. Mr. Bowman paid $628 for flour, $382 for sugar, and $125 for potatoes; how much did he receive for all if he sold them at a gain of $324? *Ans.* $1459.

11. A boy bought an arithmetic for 95 cents, a grammar for 65 cents, a history for 75 cents, a geography for $1.88, a piece of India-rubber for 5 cents, and a pencil for 3 cents; what was the whole cost? *Ans.* $4.31.

12. A merchant pays his book-keeper $1500 a year, three salesmen $750 each, a porter $450, and a boy $200; what amount of salaries does he pay? *Ans.* $4400.

13. A young man paid $450 for a horse, $115 for a sleigh, $137.50 for a harness and bells, and $33.35 for a robe; what was the cost of the whole outfit? *Ans.* $735.85.

14. At an auction a woman bid off a marble-topped bureau at $19.50, a dozen cane-seat chairs at $15.00, a carpet at $31, a small rug at $1.50, and a dozen china dinner plates at $1.75; what was her bill? *Ans.* $68.75.

15. A schooner sailed from Milwaukee to Erie, having on board 12391 bushels new No. 3 Milwaukee wheat, 11420 bushels Mixed Spring Minnesota wheat, and 1239 bushels common Winter Red Western wheat; of how many bushels did her cargo consist? *Ans.* 25050 bushels.

16. A dry goods merchant, in purchasing his stock, paid $650 for silks, $875 for woolens, $345.75 for cotton dress goods, and $795.45 for other goods; what did his stock cost? *Ans.* $2666.20.

17. Mr. Anthony's balance in bank on Monday morning was $1546.75; that day he deposited $500; Tuesday he

deposited $350; Wednesday, $764.75; Thursday, $250; Friday, $640; Saturday, $54; how much did he have on deposit on the next Monday morning? *Ans.* $4105.50.

18. A builder bought a lot for $450, built upon it a house costing $3545, and a barn and carriage-house costing $847.50, fenced it at a cost of $127.50, and graded it at a cost of $77.25; for what must he sell the property in order to gain upon it $540? *Ans.* $5587.25.

19. A Normal school paid, in 1875, $18,622.17 for salaries of professors and teachers, $1350 for salaries of other officers, $34,721.03 for board, washing, etc., $11,806.95 for servants' hire, and $7159.15 for other expenses; what were the expenditures for the year? *Ans.* $73,659.30

20. In erecting an academy, the trustees paid $250 for digging the cellar, $580 for laying the foundation walls, $12,575 for the brickwork, and $10,650 for the woodwork, plastering, etc.; what was the entire cost? *Ans.* $24,055.

21. Mr. Johnson's house cost for brick $450; for lumber, $780; other materials, $350; digging the cellar, $87.50; masons' and bricklayers' work, $425; carpenters' work, $789.75; painting, glazing, etc., $350; what was the whole cost of the house? *Ans.* $3232.25.

22. In the year ending June 1, 1875, Pennsylvania expended for school-houses, $2,059,464.83; for teachers' wages, $4,746,875.52; for fuel and contingent expenses, $2,448,315.78; for other expenses, $109,270.94; $77.324.32 extra appropriation to city of Pittsburg; $85,815.84 for Normal schools; and $423,693.76 for Soldiers' Orphan schools; what was the whole amount expended that year for education by the State? *Ans.* $9,950,760.99.

23. The cash value of farms in New York in 1870 was $1,272,857,766; in Pennsylvania, $1,043,481,582; in New Jersey, $257,523,376; in Ohio, $1,054,465,226; in Indiana, $634,804,189; in Illinois, $920,506,346; in Missouri, $392,908,047; in Wisconsin, $300,414,064; and in Michigan, $398,240,578; what is the total value of the farms in these States? *Ans* $6,275,201,174.

CONTRACTIONS IN ADDITION.

60. Contractions in Addition are abbreviated methods of adding.

61. There are several methods of abbreviating the process of addition, a few of which will be stated:

1. *In adding omit naming the numbers added; merely name the results.*

2. *When two or more terms of a column can be easily grouped together, use their sum instead of adding each separately; combining with especial reference to* TENS.

3. *When a term is repeated several times in a column, multiply it by the number of times it is repeated, and use the result.*

1. What is the sum of 8752+3687+2573+8576+2857+6872?

1ST METHOD.—We add thus: 2, 9, 15, 18, 25, 27; write the 7 and carry the 2; 2, 9, 14, 21, 28, 36, 41; write the 1 and carry the 4; etc.

2D METHOD.—Thus: 9, 15, 25, 27, in which the 3 and 7 are grouped and used as 10. We may also group 2 and 6, 7 and 3, and then 7 and 2; thus, 8, 18, 27, etc.

3D METHOD.—In the second column, we may take the *three* 7's, then the *two* 5's, and then the 8; thus, 21, 31, 39, etc.

OPERATION.

8752
3687
2573
8576
2857
6872
33317

WRITTEN EXERCISES.

(2)	(3)	(4)	(5)	(6)
67854	41235	32106	$875.67	$365.75
61387	47368	21876	321.08	406.26
57854	47376	52382	345.67	57.38
87543	51847	36875	891.01	693.84
78587	63529	42356	121.31	746.38
18234	17152	52873	415.16	25.06
34387	50381	61874	178.20	8.72
21857	27596	18027	222.32	45 50
31242	38273	25718	425.26	482.38
82356	54368	43281	272.82	29 45
14181	70305	25728	930.35	8.65
82815	75056	71890	363.73	682.50
71281	18293	12134	839.46	14.75
85427	52738	51617	876.54	937.18
18315	25382	82365	324.36	8.28
61038	32728	57634	484.95	473.54

INTRODUCTION TO SUBTRACTION.

MENTAL EXERCISES.

1. If I have 12 books and sell 5 of them, how many books shall I have remaining?

SOLUTION.—If I have 12 books and sell 5 of them, I have remaining 12 books minus 5 books, which are 7 books.

2. James had 14 cents and spent 9 of them; how many cents had he remaining?

3. Susan has 25 plums and Jane 13; how many more has Susan than Jane?

4. In a school numbering 45 pupils, 15 are absent; how many pupils are present?

5. A watch was bought for 28 dollars and sold for 21 dollars; what was the loss?

6. A cow was bought for 18 dollars and sold for 28 dollars; what was the gain?

7. Begin at 2 and count by 2's to 40; begin at 40 and count by 2's backward to 2.

8. Begin at 45 and count by 3's backward to 3; begin at 44 and count by 3's backward to 2.

9. Count by 4's from 48 back to 4; from 55 back to 3; from 54 back to 2; from 53 back to 1.

10. Count by 5's from 60 back to 5; from 64 back to 4; from 63 back to 3; from 62 back to 2; from 61 back to 1.

11. In a similar manner begin at different numbers, and count backward by 6's, 7's, 8's and 9's.

12. Take the number 3, add 5, subtract 6, add 7, subtract 5, add 8, subtract 7, add 9, subtract 4, and name the result.

13. Take the number 11, add 4, subtract 5, add 5, subtract 4, add 6, subtract 5, add 7, subtract 6, add 8, subtract 7, and name the result

14. How many are 3 plus 5 minus 7? 4 plus 7 minus 8? 5 plus 6 minus 4? 8 plus 5 minus 9? 9 plus 10 minus 12? 12 plus 13 minus 15?

15. How many are 8 plus 12 minus 13? 9 plus 16 minus 14? 10 plus 15 minus 16? 16 plus 17 minus 18? 18 plus 19 minus 20?

The process of finding the *difference* between two numbers is called *subtraction*. The sign of subtraction is —, and is read *minus*.

16. Required the value of the following:

5+3—2	2+5—4	9+8—6	10+ 7—12	13+14—15
6+4—3	3+6—5	8+6—7	11+ 8—13	14+15—17
7+2—4	4+7—8	7+5—4	12+ 7— 9	15+18—19
8+5—6	6+8—7	6+8—3	8+14—10	16+17—12
9+3—5	8+7—9	5+9—4	9+15—16	17+19—20

SUBTRACTION.

62. Subtraction is the process of finding the *difference* between two numbers.

63. The **Difference** between two numbers is a number which added to the less, equals the greater.

64. The **Minuend** is the number from which we subtract. The **Subtrahend** is the number to be subtracted.

65. The **Sign of Subtraction** is —, and is read *minus*. It denotes that the number immediately following it is to be subtracted from the number preceding it.

NOTES 1. The *Sign of Subtraction* is a short line in the line of writing.

2. The symbol — was introduced by *Stifelius*, a German mathematician, in a work published in 1544.

PRINCIPLES.

1. *The minuend and subtrahend must be similar numbers.*

2. *The difference is a number similar to the minuend and subtrahend.*

CASE I.

66. ***To subtract when no term of the subtrahend is greater than the corresponding term of the minuend.***

1. What is the difference between 486 and 243?

SOLUTION.—We write the subtrahend under the minuend, placing terms of the same order in the same column, draw a line beneath, and begin at the right to subtract. 3 units from 6 units leave 3 units, which we write under the units; 4 tens from 8 tens leave 4 tens, which we write under the tens; 2 hundreds from 4 hundreds leave 2 hundreds, which we write under the hundreds. Therefore, the difference between 486 and 243 is 243.

OPERATION.

	486
	243
	243

Rule.—I. *Write the subtrahend under the minuend, placing terms of the same order in the same column, and draw a line beneath.*

II. *Begin at the right and subtract each term of the subtrahend from the corresponding term of the minuend, writing the remainder beneath.*

WRITTEN EXERCISES.

	(2)	(3)	(4)	(5)
From	364	487	524	876
Subtract	123	235	321	235

	(6)	(7)	(8)	(9)
From	387	369	879	872
Subtract	215	123	354	351

	(10)	(11)	(12)	(13)
From	895	718	$38.57	$51.83
Subtract	713	315	12.35	21.62

	(14)	(15)	(16)	(17)
From	8917	8975	$37.59	$75.68
Subtract	5302	3623	25.36	34.15

	(18)	(19)	(20)	(21)
From	6753	8917	7369	$98.76
Subtract	5241	7214	2134	54.32

22. Subtract 34512 from 67856. *Ans.* 33344.
23. Subtract 41231 from 81275. *Ans.* 40044.
24. Subtract 32125 from 96576. *Ans* 64451.
25. Subtract 14114 from 85237. *Ans.* 71123
26. Subtract 23121 from 64875. *Ans.* 41754.
27. Subtract 23254 from 48796. *Ans.* 25542.
28. Subtract $123.42 from $9876.54. *Ans.* $9753.12.
29. Subtract $2413.25 from $7654.87. *Ans.* $5241.62.

CASE II.

67. ***To subtract when one or more terms of the subtrahend are greater than the corresponding terms of the minuend.***

68. There are two methods of explaining this case, called the *Method of Borrowing* and the *Method of Adding Ten.* We will solve the same problem by both methods.

1. From 836 subtract 472.

SOLUTION BY BORROWING.—We write the subtrahend under the minuend, and begin at the right to subtract. 2 units from 6 units leave 4 units, which we write under the units; we cannot take 7 tens from 3 tens, we will therefore take 1 hundred from the 8 hundreds, and add it to the 3 tens; 1 hundred equals 10 tens, which, added to 3 tens, equals 13 tens; 7 tens from 13 tens leave 6 tens, which we write in tens place; 4 hundreds from 7 hundreds (the number remaining after taking away 1 hundred) leave 3 hundreds, which we write in the hundreds place.

OPERATION.

836
472
364

SOLUTION BY ADDING TEN.—2 units from 6 units leave 4 units; we cannot take 7 tens from 3 tens, we will therefore add 10 tens to the 3 tens, making 13 tens; 7 tens from 13 tens leave 6 tens; now, since we have added 10 tens, or 1 hundred, to the minuend, our remainder will be 1 hundred too large; hence to obtain the correct remainder we must add 1 hundred to the subtrahend; 1 hundred and 4 hundreds are 5 hundreds; 5 hundreds from 8 hundreds leave 3 hundreds.

Rule.—I. *Write the subtrahend under the minuend, placing terms of the same order in the same column, and draw a line beneath.*

II. *Begin at the right and subtract each term of the subtrahend from the corresponding term of the minuend, writing the remainder beneath.*

III. *If any term of the subtrahend is greater than the corresponding term of the minuend, add* 10 *to the latter, and then subtract.*

IV. *Add* 1 *to the next term of the subtrahend (or subtract* 1 *from the next term of the minuend), and proceed as before.*

Proof.—Add the difference to the subtrahend; and if the work is correct the sum will equal the minuend.

SECOND METHOD.—Subtract the difference from the minuend, and if the work is correct, the result will equal the subtrahend.

NOTE.—The taking one from a term of the minuend is called *borrowing*, and the adding one to the next term of the subtrahend is called *carrying*.

WRITTEN EXERCISES.

	(2)	(3)	(4)	(5)
From	386	462	$5.23	$6.15
Take	157	175	1.46	1.47
	(6)	(7)	(8)	(9)
From	3123	6357	7518	4712
Take	1415	2829	2036	3508
	(10)	(11)	(12)	(13)
From	4075	3987	3013	6514
Take	2867	1989	2187	2823
	(14)	(15)	(16)	(17)
From	$571.23	$605.32	$657.00	$451.00
Take	278.51	518.18	472.83	132.18

18. From 94278 subtract 62573. *Ans.* 31705.
19. From 70532 subtract 25824. *Ans.* 44708.
20. From 53715 subtract 15186. *Ans.* 38529.
21. From 80526 subtract 25367. *Ans.* 55159.
22. From 75138 subtract 61859. *Ans.* 13279.
23. From \$200.87 subtract \$123.48. *Ans.* \$77.39.

What is the value

24. Of 12857+3659—2768? *Ans.* 13748.
25. Of 34821+3127—5879? *Ans.* 32069.
26. Of 8000732—5001916? *Ans.* 2998816.
27. Of 1000000+10000—1? *Ans.* 1009999.
28. Of 8032—6257+18765? *Ans.* 20540.
29. Of 2875—2382+67876? *Ans.* 68369.
30. Of \$2.85—\$1.86+\$3.92—\$2.56? *Ans.* \$2.35.
31. From ten thousand take nine hundred and ninety-nine. *Ans.* 9001.
32. From one million and one, take three thousand and three. *Ans.* 996998.
33. From four hundred thousand take four hundred and forty-four. *Ans.* 399556.
34. From seventy-seven thousand take seven thousand and seventy-seven. *Ans.* 69923.
35. From one hundred million and one take one million and five. *Ans.* 98999996.
36. From 6 billion, 6 million, 6 thousand and 6, take 80 million, 80 thousand and 8. *Ans.* 5925925998.

MENTAL EXERCISES.

1. Mary bought 20 apples and gave Ella 6 of them; how many did Mary then have?

SOLUTION.—If Mary bought 20 apples and gave Ella 6 of them, Mary then had the difference between 20 apples and 6 apples, which is 14 apples.

2. A boy found 16 eggs in the barn and broke 8 of them; how many remained unbroken?

3. How many birds will escape if I shoot 6 birds out of a flock of 40 birds?

4. How many are 17 less 8? 14 less 9? 21 less 7? 23 less 6? 25 less 8? 31 less 4?

5. How many are 32 less 3? 41 less 2? 50 less 6? 49 less 7? 78 less 8? 100 less 9? 124 less 12?

6. Required the value of 112 minus 8; 120 minus 7; 125 minus 8; 132 minus 5; 140 minus 4; 150 minus 5; 150 minus 10.

7. A boy having 12 apples, bought 6 more, and then sold 8; how many had he then?

8. Mary having 14 cents, spent 8 cents, after which her father gave her 12 cents; how many cents had she then?

9. Edwin had $24 and earned $6, and then spent $12; how much did he then have?

10. A merchant gave $65 for goods, and paid $6 for transportation; at what price must he sell the goods to gain $12?

11. Subtract 36 from 84.

SOLUTION.—30 from 84 leaves 54, and 6 from 54 leaves 48.

12. Subtract 26 from 42; 35 from 52; 38 from 60; 42 from 75; 45 from 72; 48 from 84; 52 from 86; 56 from 82.

13. Subtract 123 from 147; 135 from 248; 152 from 285; 206 from 456; 307 from 444; 328 from 565.

14. In a school numbering 75 pupils 28 are absent; how many pupils are present?

15. A lady bought a watch for $250 and sold it at a loss of $35; what did she receive for it?

16. A man bought a horse for $175, paid $15 for his keeping, and sold him for $240; what was the gain?

17. A lady bought some needles for 8 cents, some thread for 15 cents, and some tape for 18 cents, and hands the clerk a 50-cent note; what is her change?

18. A man went into a clothing store and bought a vest for $6, a coat for $25, and a pair of pants for $12; he hands the clerk 5 ten dollar bills; what change will he receive?

19. Count by 6's from 6 to 72 and back; from 5 to 77 and back from 4 to 76 and back; from 3 to 75 and back; etc.

20. Count by 7's from 7 to 84 and back; from 6 to 90 and back from 5 to 89 and back; from 4 to 88 and back; etc.

21. Count by 8's from 0 to 96 and back; from 1 to 97 and back from 2 to 98 and back; from 3 to 99 and back; etc.

22. Count by 9's from 0 to 108 and back; from 1 to 109 and back from 2 to 110 and back; from 4 to 112 and back; etc.

23. Take the number 12, add 3, subtract 2, add 4, subtract 3, add 5, subtract 4, add 6, subtract 5, add 7, subtract 6, and name the result.

24. How many are 12 plus 8, minus 7, plus 10, minus 6, plus 12, minus 9, plus 14, minus 18, plus 25, minus 16?

25. How many are 25 plus 20, minus 16, plus 36, minus 40, plus 56, minus 25, plus 72, minus 84, plus 120, minus 140?

WRITTEN EXERCISES.

1. A man had 725 bushels of wheat and sold 367 bushels; how many bushels remained?

SOLUTION.—If a man had 725 bushels of wheat and sold 367 bushels, there remained the difference between 725 bushels and 367 bushels, which is 358 bushels.

OPERATION.

725
367
358

2. A farmer raised 627 bushels of potatoes, and sold 386 bushels; how many bushels remained? *Ans.* 241 bushels.

3. From a farm containing 3075 acres, 2528 acres were sold; how many acres remained? *Ans.* 547 acres.

4. A man borrowed $3052 and afterward returned $2527; how much does he still owe? *Ans.* $525.

5. A sold his house for $2572, which was $285 more than it cost; how much did it cost? *Ans.* $2287.

6. Two men together have $5682; how much money has the first if the second has $3826? *Ans.* $1856.

7. A man bought a farm for $10,852, and sold it at a loss of $987; what did he receive for it? *Ans.* $9865.

8. A and B together own 10,840 acres of land; how much does B own if A owns 5284 acres? *Ans.* 5556 acres.

9. Gen. Zachary Taylor was born in 1790 and died in 1850; how old was he when he died? *Ans.* 60 years.

10. Robert Southey was born at Bristol in 1774 and died in 1843; how old was he at his death? *Ans.* 69 years.

11. Socrates was born at Athens 468 years before Christ, and died 398 years before Christ; what was his age at death? *Ans.* 70 years.

12. Walter Scott was born in the year 1771, and died in 1832; how old was he at his death? *Ans.* 61 years.

13. How many years from the birth of Milton in 1608 to the birth of George Washington in 1732? *Ans.* 124 years.

14. A farmer having 478 cows, sold 198 of them, and then bought 226; how many had he then? *Ans.* 506.

15. How many years from the birth of William Penn in 1644 to the birth of Benjamin Franklin in 1706?

Ans. 62 years.

16. A merchant having 336 pounds of rice, bought 248 pounds, and then sold 467 pounds; how many pounds remained? *Ans.* 117 pounds.

17. Lanterns were invented by King Alfred in 890; how many years from that time to 1876? *Ans.* 986 years.

18. Spectacles were invented by Spina in 1299; how many years from their invention to 1878? *Ans.* 579 years.

19. Watches were invented at Nuremberg in 1477; ho many years from that time to 1876? *Ans.* 399 years.

20. The circulation of the blood was discovered by Harvey in 1619; how long since that time?

21. The telescope was invented by Galileo in 1610; how many years since it was invented?

22. Printing is supposed to have been invented in 1441; how many years has the art been used?

23. The first steamship crossed the ocean in 1839; how long is it since that time?

24. The first line of telegraph was established in the United States in 1844; how long has the invention been in use?

25. The distance from Philadelphia to Pittsburgh is 353 miles, and to Harrisburg is 105 miles; what is the distance from Harrisburg to Pittsburgh? *Ans.* 248 miles.

26. A merchant had in bank $4500, and "checked out" $2345; how much will remain in bank? *Ans.* $2155.

27. An architect received $54,750 for building a market-house, and he expended upon it $51,784; what was his profit? *Ans.* $2966.

28. Mr. Newton bought a house for $10,575, and paid for it in yearly installments of $1057.50; how many years did it require to pay for the house? *Ans.* 10 years.

29. At an election the successful candidate received 40.172 votes, and the unsuccessful candidate 34,789 votes; what was the successful candidate's majority?

Ans. 5383 votes.

CONTRACTIONS IN SUBTRACTION.

69. Contractions in Subtraction are abbreviated methods of subtracting.

70. When two or more numbers are to be successively subtracted from another, the operation can be abbreviated by the following rule:

Rule.—*Add the terms of the numbers to be subtracted, and write for a remainder a term which, added to this sum, will give a number having for its unit term the corresponding term of the minuend.*

1. Subtract 324 and 549 from 1024.

SOLUTION.—4 and 9 are 13, and since 1 more will make 14, we write 1 as the first term of the remainder; 4 and 1 are 5 and 2 are 7, and since 5 more make 12, we write 5 in the remainder; 5 and 1 are 6 and 3 are 9, and since 1 more make 10, we write 1 in the remainder.

OPERATION.

1024
324
549
151

WRITTEN EXERCISES.

(2)	(3)	(4)	(5)
5765	85741	69547	1000000
1334	13333	14937	676543
1562	14792	18356	135792
	10234	3344	187665

6. A raised 785 bushels of grain, and having sold B 250 bushels, C 320 bushels, and D 169 bushels, he retained the remainder; how many bushels did he retain? *Ans.* 46 bu.

7. From a tract of land in a newly settled country containing 4850 acres, there were sold at one time 1560 acres; at another, 2490 acres; and at another 500 acres; how much of the tract then remained? *Ans.* 300 acres.

8. In five granaries there are 7874 bushels of wheat; in the first there are 1160 bushels; in the second 1729 bushels; in the third 1957 bushels; in the fourth 1433 bushels; how many bushels are contained in the fifth? *Ans.* 1595 bu.

9. An estate of $10,000 was divided as follows: the widow received $4397; the youngest son, $2498; the daughter, $3104; and the eldest son the remainder. What was the share of the latter?

WRITTEN EXERCISES.

IN ADDITION AND SUBTRACTION.

1. An estate worth $35,000 has two mortgages upon it, one for $15,000, and the other for $8000; what is it worth above the incumbrances? *Ans.* $12,000.

2. A merchant's gross profits last year were $9000; he pays for rent $1200, insurance $150, salaries $3500, and incidental expenses, $225; what is his net profit? *Ans.* $3925.

3. A clerk receives $800 a year; he pays $150 a year for house rent, his butcher's bill is $177.50 and grocer's bill $145.45, and he has expended for clothes and other expenses $250; what does he save in the year? *Ans.* $77.05.

4. Mr. Wainwright bought a farm which cost him $25,000; but afterwards getting into difficulties, he burdened the property with a mortgage for $7500; for what must he sell the farm to gain $3500? *Ans.* $21,000.

5. Mr. Hatton receives for goods sold a check for $15,495.25 on the Girard National Bank; he wishes to deposit $13,500 in the same bank; what will the bank return him when he presents the check? *Ans.* $1995.25.

6. I had in bank on Monday morning, Dec. 4, $11,275, I checked out Monday afternoon $745; Tuesday, I deposited $1500; Wednesday, I deposited $475; Thursday, I checked out $873.50; Friday, I again checked out $563.75; Saturday, I deposited $560; what is the amount of my deposit on Monday morning, Dec. 11? *Ans.* $11,627.75.

7. The day before Christmas I went into a fancy store and bought for Christmas presents, a writing-desk at $4.25, a work-box at $2.75, a glove-box at $1.50, a handkerchief-box at $1.75, a case of perfumery at $2.25, and a gentleman's dressing-case at $5.75; I handed the clerk a twenty-dollar bill; what change should I receive? *Ans.* $1.75.

8. A farmer took to the store 2 dozen eggs, $0.70; 10 pounds of butter, $5; 2 barrels of apples, $11.50; he bought 25 pounds crushed sugar, $3.50; 2 gallons molasses, $0.60; 10 yards red flannel, $3.75, and one piece unbleached sheeting, $10; how much cash must he pay? *Ans.* $0.65.

INTRODUCTION TO MULTIPLICATION.

MENTAL EXERCISES.

1. Repeat the table of 2 times; of 3 times; of 4 times; of 5 times of 6 times; of 7 times; of 8 times; etc.

2. If one barrel of flour costs 6 dollars, what will 5 barrels cost at the same rate?

SOLUTION.—If one barrel of flour costs 6 dollars, 5 barrels will cost 5 times 6 dollars, which are 30 dollars.

3. What will 7 gold pens cost at 3 dollars apiece?

4. At 20 cents apiece what will 6 spelling-books cost?

5. What is the cost of 11 yards of calico at 12 cents a yard?

6. George has 15 dollars and James has 6 times as much; how many dollars has James?

7. A farmer bought 6 cows at 25 dollars apiece; how much did he pay for them?

8. If 6 men can do a piece of work in 20 days, how long will it take 1 man to do it?

9. If 8 men mow a field of grass in 15 days, how long will it take 1 man to mow it?

10. In an orchard there are 16 rows of trees and 10 trees in each row; how many trees are there in the orchard?

11. John borrowed 12 cents from a friend, and then earned 6 times as much as he borrowed; how many cents had he then?

12. Arnold earned 18 dollars a week, and paid 9 dollars a week for his board; how much could he save in 8 weeks?

13. How many are 3 times 5, plus 6? 5 times 6, plus 7? 6 times 7, plus 8? 7 times 8, plus 9? 9 times 10, plus 11? 12 times 10, plus 13?

14. How many are 4 times 6, minus 8? 6 times 7, minus 5? 8 times 9, minus 10? 7 times 11, minus 9? 9 times 12, minus 13?

15. How many are 6 and 6 times 7? 7 and 7 times 9? 8 and 8 times 10? 9 and 9 times 8? 10 and 10 times 11? 11 and 11 times 12?

16. The process of taking one number as many times as there are units in another is called *multiplication.* The number multiplied is the *multiplicand;* the number by which we multiply is the *multiplier;* the result is the *product.* The sign of multiplication is $\times$, and is read "multiplied by."

17. Read the result of the following:

$5\times4+8$	$7\times3+2$	$9\times7+12$	$12\times9-18$	$12\times12-24$
$6\times5+4$	$6\times4-5$	$8\times6-11$	$10\times8+16$	$13\times\ 6-18$
$7\times6+5$	$5\times5+7$	$7\times8-10$	$11\times4-15$	$14\times\ 7-27$
$8\times7+6$	$4\times6-8$	$8\times4+12$	$13\times3-\ 9$	$16\times\ 5-30$
$9\times8+7$	$3\times7+5$	$9\times3-10$	$14\times4-12$	$17\times\ 3-21$

MULTIPLICATION.

71. Multiplication is the process of finding the *product* of two numbers.

72. The **Product** of two numbers is the result obtained by taking one number as many times as there are units in another.

73. The **Multiplicand** is the number to be multiplied.

74. The **Multiplier** is the number by which we multiply.

75. The **Sign of Multiplication** is ×, and is read *multiplied by*, *times*, or *into*. When placed between two numbers it denotes that one is to be multiplied by the other.

NOTES.—1. The *Sign of Multiplication* consists of two short lines of equal length, bisecting each other at an angle of 45 degrees with the line of writing.

2. The symbol × was introduced by *Wm. Oughtred*, an English mathematician, born in 1574.

PRINCIPLES.

1. *The multiplier is always an abstract number.*
2. *The multiplicand may be abstract or concrete.*
3. *The product is always similar to the multiplicand.*

CASE I.

76. ***When the multiplier is not greater than twelve.***

1. Multiply 353 by 8.

SOLUTION.—We write the multiplier under the multiplicand, draw a line beneath, and begin at the right to multiply. 8 times 4 units are 32 units, or 2 units and 3 tens; we write the units in units place, and reserve the tens to add to the next product: 8 times 5 tens are 40 tens, plus the 3 tens equal 43 tens, or 3 tens and 4 hundreds; we write the 3 tens in tens place, and reserve the 4 hundreds to add to the next product: 8 times 3 hundreds are 24 hundreds, plus the 4 hundreds equal 28 hundreds, or 8 hundreds and 2 thousands, which we write in their proper places, and we have 2832. Hence the following

OPERATION.

Multiplicand,	354
Multiplier,	8
Product,	2832

Rule.—I. *Write the multiplier under the multiplicand, and draw a line beneath.*

II. *Begin at the right, and multiply each term of the multiplicand by the multiplier, carrying as in addition.*

WRITTEN EXERCISES.

(2)	(3)	(4)	(5)
567	682	795	821
3	4	2	7
1701	2728	1590	5747

(6)	(7)	(8)	(9)
$3.07	$5.86	$12.58	$25.08
8	6	5	9
$24.56	$35.16	$62.90	$225.72

Multiply

10. 5708 by 8.	*Ans.* 45664.	16. 32187 by 3.	*Ans.* 96561.
11. 3218 by 7.	*Ans.* 22526.	17. 58799 by 6.	*Ans.* 352794.
12. 2107 by 6.	*Ans.* 12642.	18. 25767 by 5.	*Ans.* 128835.
13. 5189 by 7.	*Ans.* 36323.	19. 41937 by 4.	*Ans.* 167748.
14. 12076 by 4.	*Ans.* 48304.	20. 82709 by 8.	*Ans.* 661672.
15. 21816 by 5.	*Ans.* 109380.	21. 70095 by 9.	*Ans.* 630855.

22. Multiply 97548 by 9. *Ans.* 877932.

23. Multiply 847946 by 9. *Ans.* 7631514.

24. If one acre of land is worth 275 dollars, what are 7 acres worth at the same rate?

SOLUTION.—If one acre of land is worth 275 dollars, 7 acres are worth 7 times 275 dollars, which we find by multiplying is 1925 dollars. Therefore, etc.

OPERATION.

275
7
1925 *Ans.*

25. Sound moves 1120 feet in a second; how far will it move in six seconds? *Ans.* 6720 feet.

26. If a horse cost $150, what will 8 horses cost at the same rate? *Ans.* $1200.

27. If there are 1375 letters on one page of a book, how many letters on 7 pages at the same rate? *Ans.* 9625.

28. If there are 231 cubic inches in one gallon, how many cubic inches in five gallons? *Ans.* 1155.

29. Light moves about 192,500 miles a second; how far will it move in four seconds? *Ans.* 770,000.

30. The distance of the moon from the earth is about 240,000 miles; what is nine times the distance?

Ans. 2,160,000.

CASE II.

77. ***When the multiplier is greater than twelve.***

1. Multiply 465 by 37.

SOLUTION.—We write the multiplier under the multiplicand, units under units, tens under tens, etc. Since 37 equals 7 units and 3 tens, it is evident that 37 times a number equals 7 times the number plus 3 tens times the number. Seven times 465 equals 3255; 3 times 465 equals 1395, hence 3 tens times 465 equals 1395 tens. Taking the sum of the partial products we have 17205. Hence, etc.

OPERATION.

465
37
3255
1395
17205

Rule.—I. *Write the multiplier under the multiplicand, placing terms of the same order in the same column, and draw a line beneath.*

II. *Begin at the right, and multiply the multiplicand by each term of the multiplier, writing the first term of each product under the term of the multiplier which produces it.*

III. *Add the partial products, and their sum will be the entire product.*

Proof.—Multiply the multiplier by the multiplicand; if the work is correct this product will equal the first product.

NOTES.—1. When there are ciphers between the significant terms of the multiplier, pass over them and multiply by the significant terms alone.

2. We begin at the right to multiply, so that when any product exceeds nine we may add the number expressed by the left hand figure to the next product.

2. Multiply 247 by 24; also by 204.

OPERATION.

247
24
988
494
5928

OPERATION.

247
204
988
494
50388

WRITTEN EXERCISES.

(3)	(4)	(5)	(6)
354	234	314	321
14	15	16	18
4956	3510	5024	5778
(7)	**(8)**	**(9)**	**(10)**
$4.16	$4.17	$2.45	$2.18
21	23	24	25
$87.36	$95.91	$58.80	$54.50

11. Multiply 372 by 26. *Ans.* 9672.
12. Multiply 274 by 29. *Ans.* 7946.
13. Multiply 418 by 27. *Ans.* 11286.
14. Multiply 625 by 28 *Ans.* 17500.
15. Multiply 408 by 34. *Ans.* 13872.
16. Multiply 316 by 35. *Ans.* 11060.
17. Multiply 683 by 36. *Ans.* 24588.
18. Multiply 672 by 38. *Ans.* 25536
19. Multiply 811 by 39. *Ans.* 31629.
20. Multiply 725 by 41. *Ans.* 29725.
21. Multiply 437 by 43. *Ans.* 18791.
22. Multiply 1621 by 45 *Ans.* 72945.
23. Multiply 1763 by 46. *Ans.* 81098.
24. Multiply 1234 by 49. *Ans.* 60466.
25. Multiply 6171 by 54. *Ans.* 333234.
26. Multiply 1876 by 56. *Ans.* 105056.
27. Multiply 2181 by 58. *Ans.* 126498.
28. Multiply 2931 by 59. *Ans.* 172929.
29. Multiply 3082 by 68. *Ans.* 209576.
30. Multiply 4107 by 93. *Ans.* 381951.
31. Multiply 2573 by 76. *Ans.* 195548.
32. Multiply 3871 by 87. *Ans.* 336777.
33. Multiply 5482 by 135. *Ans.* 740070.
34. Multiply 3257 by 246. *Ans.* 801222
35. Multiply 4185 by 368. *Ans.* 1540080.
36. Multiply 5682 by 543. *Ans.* 3085326.
37. Multiply 7328 by 1021. *Ans.* 7481888.
38. Multiply 2567 by 2036. *Ans.* 5226412.
39. Multiply 2135 by 3007. *Ans.* 6419945.
40. Multiply 6328 by 1276. *Ans.* 8074528.
41. Multiply 5682 by 1083. *Ans.* 6153606.
42. Multiply 4185 by 1472. *Ans.* 6160320.
43. Multiply 3257 by 1576. *Ans.* 5133032.
44. Multiply 7328 by 2042. *Ans.* 14963776.
45. Multiply 2567 by 4072. *Ans.* 10452824
46. Multiply 2135 by 6014. *Ans.* 12839890.
47. Multiply 71873 by 3018. *Ans.* 216912714.

48. Multiply 6328 by 2552.	*Ans.* 16149056.
49. Multiply 71873 by 6036.	*Ans.* 433825428.
50. Multiply 11364 by 1629.	*Ans.* 18511956
51. Multiply 21984 by 1021.	*Ans.* 22445664.
52. Multiply 12835 by 2036.	*Ans.* 26132060.
53. Multiply 10675 by 3007.	*Ans.* 32099725.
54. Multiply 25312 by 1276	*Ans.* 32298112.
55. Multiply 25312 by 2552.	*Ans.* 64596224.
56. Multiply 57873 by 4812.	*Ans.* 278484876.
57. Multiply 73218 by 4167.	*Ans.* 305099406.
58. Multiply 35821 by 3782.	*Ans.* 135475022.
59. Multiply 85765 by 81072.	*Ans.* 6953140080.
60. Multiply 180071 by 23456.	*Ans.* 4223745376.
61. Multiply 871352 by 40709.	*Ans.* 35471868568
62. Multiply 826107 by 57018.	*Ans.* 47102968926.
63. Multiply 360073 by 360073.	*Ans.* 129652565329.
64. Multiply 473009 by 874030.	*Ans.* 413424056270.

MENTAL EXERCISES.

1. What will 6 yards of muslin cost at 10 cents a yard?

SOLUTION.—If 1 yard of muslin costs 10 cents, 6 yards of muslin will cost 6 times 10 cents, which are 60 cents.

2. What will 8 sheep cost at 12 dollars apiece?

3. What will 12 turkeys cost at 60 cents apiece?

4. At 80 cents each, how much will 9 arithmetics cost?

5. If 5 men can do a piece of work in 6 days, how long will it take 1 man to do it?

6. If 9 men dig a trench in 14 days, how long would it take one man to do it?

7. How many trees in an orchard in which there are 12 rows with 20 trees in each row?

8. A grocer had 24 melons, sold 10 of them, and then bought 5 times as many as he sold; how many had he then?

9. Mason earned $15 a week and paid $7 for his board; how much could he save in 8 weeks?

10. Thompson earned $50 a month and paid $5 a week for his board, and $4 for other expenses; how much could he save in a year?

11. Multiply 76 by 4.

SOLUTION.—4 times 6 units are 24 units, which equals 2 *tens* and 4 *units*, 4 times 7 *tens* are 28 *tens*, which increased by the 2 tens equals 30 *tens*; hence the product is 304.

12. Multiply 26 by 3; 28 by 4; 37 by 5; 46 by 6; 53 by 7; 75 by 8; 65 by 12; 88 by 11; 125 by 10.

13. Multiply 142 by 4; 325 by 3; 463 by 5; 372 by 6; 408 by 7; 1234 by 6; 2007 by 8; 1306 by 5; 508 by 11; 407 by 12.

14. If a railroad train runs 36 miles an hour, how far will it run at that rate in 6 hours?

15. If a newsboy clears 56 cents a day for 8 days, how much will he clear in all?

16. A jeweler bought 8 watches for $125 apiece, and sold them at a gain of $25 each; what did he receive for them?

17. At $6 a barrel, how much are 3 loads of flour worth, each load containing 10 barrels?

18. A man bought 5 books at 40 cents each, and 2 pairs of shoes at $1.50 a pair; what change will be left out of a $5 bill?

19. A farmer's wife took to a store 6 dozen eggs at 20 cents a dozen, and bought 8 yards of muslin at 12 cents a yard; what balance should she receive?

20. A lady went "a shopping" and bought 6 yards of silk at $3 a yard, 2 yards of velvet at $5 a yard, and 4 pairs of hose at 75 cents a pair; she handed the clerk two $20 bills; what was her change?

WRITTEN EXERCISES.

1. If a steamer sails 235 miles in 1 day, how far will she sail in 37 days?

SOLUTION.—If a steamer sails 235 miles in 1 day, in 37 days she will sail 37 times 235 miles, or 8695 miles.

OPERATION.

235
37
1645
705
8695

2. How many bushels of potatoes can be raised on 56 acres of land, if each acre produces 196 bushels? *Ans.* 10,976.

3. How many oranges in 59 boxes, if each box contains 275 oranges? *Ans.* 16,225 oranges.

4. If a clerk deposits $575 annually in a savings bank, how much will he deposit in 25 years? *Ans.* $14,375.

5. Mr. Gibbs bought a farm of 95 acres at $187 an acre; what was the cost of the farm? *Ans.* $17,765.

6. What will a hogshead of wine, containing 63 gallons, cost at the rate of $4.25 a gallon? *Ans.* $267.75.

7. I bought 356 sacks of Liverpool salt at $2.50 per sack; what did I pay for the whole amount? *Ans.* $890.

8. Sold 265 barrels of mess pork at $16.50 a barrel; what did I receive for the whole? *Ans.* $4372.50.

9. If a locomotive-wheel revolves 478 times in going 1 mile, how often will it revolve in 248 miles? *Ans.* 118,544.

10. When the market-price of butter is 18 cents a pound, what must I pay for 256 pounds? *Ans.* $46.08.

NOTE.—In this and similar problems we multiply by the smaller number, using both numbers abstractly, though in the explanation we should use the proper number as the multiplier.

11. If a barrel of flour is worth $8, how much are 596 barrels worth at the same rate? *Ans.* $4768.

12. A dealer bought 2875 bushels of potatoes at 37 cents a bushel; what was the cost? *Ans.* $1063.75.

13. A Western farmer sold 9876 bushels of wheat at 85 cents a bushel; what did he receive? *Ans.* $8394.60.

14. Henry earned $28 a week and paid $5 for his board; how much could he save in 52 weeks? *Ans.* $1196.

15. A travels 25 miles a day, and B 36 miles a day; how far will both travel in 286 days? *Ans.* 17,446 miles.

16. The President's cabinet consists of 7 members, who receive a salary of $8000 each; what is the amount of their salaries? *Ans.* $56,000.

17. A dealer shipped 2396 bushels of apples at 87 cents a bushel, and the same quantity of potatoes at 56 cents a bushel; what was the value of both? *Ans.* $3426.28.

18. A bought 2960 acres of prairie land at $38 an acre, and the same number of acres of woodland at $45 an acre; what did he pay for all? *Ans.* $245,680.

19. In a row of houses there are 46 rooms, in each room 4 windows, and in each window 12 panes of glass; how many panes in all the houses? *Ans.* 2208 panes.

20. If Thomas sells 48 papers a day, and Henry 72 papers how many more papers does Henry sell in 94 days than Thomas? *Ans.* 2256 papers.

21. In 1852, 157,548 Irish immigrants arrived in the United States; at this rate what would be the number of immigrants from the beginning of 1852 to the end of 1875? *Ans.* 3,781,152.

22. If two persons should start from the same place and travel in opposite directions, one at the rate of 45 miles a day and the other at the rate of 67 miles a day, how far will they be apart in 67 days? *Ans.* 7504 miles.

23. A planter put up his cotton into 275 bales, averaging 576 pounds each, and sold it at 15 cents per pound; what did he receive for the whole? *Ans.* $23,760.

24. Mr. Angell's annual income is $9750, and his average daily expenditure is $9.75; what can he save in a year of 365 days? *Ans.* $6191.25.

25. A bank teller receives $2500 salary, and spends $520 for board, $475 for clothing, $112.65 for books, and $117.25 for other expenses annually; how much will he save in 5 years? *Ans.* $6375.50.

26. How many miles will a railroad conductor travel in a year if he makes a trip from Harrisburg to Philadelphia (106 miles) and back, every day except Sundays, reckoning 52 Sundays to a year? *Ans.* 66,356 miles.

27. A shoe-dealer bought 35 cases of French calf boots, each case containing 12 pairs, at $6.50 a pair; what did the bill amount to? *Ans.* $2730.

28. A division consisted of 4 regiments, each regiment of 9 companies, and each company of 98 men; how many men in the division? *Ans.* 3528 men.

29. In a block of houses there are 45 buildings, each building containing 32 windows, and each window 12 panes of glass; how many panes of glass in the whole block of buildings? *Ans.* 17,280 panes.

30. If it requires 124 tons of iron worth $72 a ton to build one mile of a railroad, what will be the cost of iron enough to build a railroad 236 miles long? *Ans.* $2,107,008.

31. A field contains 5076 rows of corn, each row containing 4005 hills, and each hill 4 stalks; how many stalks of corn in the field? *Ans.* 81,317,520 stalks.

CONTRACTIONS IN MULTIPLICATION.

78. Contractions in Multiplication are abbreviated methods of multiplying.

79. A Composite Number is the product of two or more numbers, each greater than a unit, called *factors*. Thus, 24 is a composite number, whose factors are 4 and 6, or 3 and 8, or 2, 3, and 4.

CASE I.

80. ***When the multiplier is a composite number.***

1. Multiply 53 by 24.

SOLUTION.—24 equals 4 times 6, hence 24 times 53 equals 4 times 6 times 53. 6 times 53 equals 318, and 4 times 6 times 53 equals 4 times 318, which equals 1272; therefore, 53 multiplied by 24 equals 1272. Hence the

OPERATION

53
6
318
4
1272

Rule.—*Multiply the multiplicand by one factor, this product by another factor, and thus continue until all the factors have been used; the last product will be the result required.*

Multiply

2. 85 by 35.	*Ans.* 2975.	**8.** 362 by 45.	*Ans.* 16290.
3. 98 by 16.	*Ans.* 1568.	**9.** 893 by 49.	*Ans.* 43757.
4. 75 by 72.	*Ans.* 5400.	**10.** 3572 by 35.	*Ans.* 125020.
5. 87 by 56.	*Ans.* 4872.	**11.** 4087 by 27.	*Ans.* 110349.
6. 123 by 63.	*Ans.* 7749.	**12.** 8937 by 42.	*Ans.* 375354.
7. 248 by 54.	*Ans.* 13392.	**13.** 40729 by 72.	*Ans.* 2932488.

14. What cost 35 cows at 28 dollars apiece?

SOLUTION.—35 equals 5 times 7. If one cow costs 28 dollars, 7 cows will cost 7 times 28 dollars, which are 196 dollars; and 35 cows, which are 5 times 7 cows, will cost 5 times 196 dollars, which are 980 dollars. Therefore, etc.

OPERATION.

28
7
196
5
980

15. What cost 21 yards of muslin, at 14 cents a yard?

Ans. $2.94.

16. What cost 15 barrels of fish, at $13 a barrel?

Ans. $195.

17. What cost 24 horses, at the rate of $165 each?

Ans. $3960

18. What will 42 cows cost at the rate of $27 apiece?

Ans. $1134.

19. What must I pay for 84 books, at the rate of $3.25 each? *Ans.* $273.

20. If one bushel of rye is worth $0.84, how much are 64 bushels worth? *Ans.* $53.76.

21. If a yoke of oxen cost $137, what will 56 yoke cost at the same rate? *Ans.* $7672.

22. What cost 64 acres of land, at $256 an acre; and what cost 81 acres at the same rate? *Ans.* 2d, $20,736

CASE II.

81. ***When there are ciphers at the right of one or both factors.***

Principle.—*Annexing one cipher to a number multiplies it by* 10; *annexing two ciphers multiplies it by* 100; *annexing three, multiplies it by* 1000, *etc.*

For, adding one cipher removes each term one place to the left, and thus makes it denote 10 times as many units as before, hence the entire number is ten times as great as before.

1. Multiply 26 by 140, also 2600 by 140.

SOLUTION 1ST.—26 multiplied by 14 equals 364; hence 26 multiplied by 140, which is 10 times 14, equals 10 times 364, which, by annexing one cipher, equals 3640.

SOLUTION 2D.—14 times 26 equals 364, hence 14 times 26 *hundred* equals 100 times as much, which by annexing two ciphers, is 36400; and 140 times 2600 equals 10 times 36400 which, by annexing one cipher, is 364000. Hence the following rule:

OPERATION.	OPERATION.
26	2600
140	140
104	104
26	26
3640	364000

Rule.—*Take the product of the numbers denoted by the significant figures, and annex as many ciphers to the result as are found at the right of both factors.*

WRITTEN EXERCISES.

2. Multiply 725 by 60. *Ans.* 43500.
3. Multiply 927 by 80. *Ans.* 74160.
4. Multiply 2187 by 300. *Ans.* 656100.

5. Multiply 4109 by 500. *Ans.* 2054500
6. Multiply 5090 by 700. *Ans.* 3563000.
7. Multiply 7355 by 6000. *Ans.* 44130000.
8. Multiply 2170 by 9000. *Ans.* 19530000.
9. Multiply 8542 by 2500. *Ans.* 21355000.
10. Multiply 7681 by 7300. *Ans.* 56071300.
11. Multiply 6500 by 2500. *Ans.* 16250000.
12. Multiply 7800 by 6700. *Ans.* 52260000.
13. Multiply 35600 by 12500. *Ans.* 445000000.
14. Multiply 87200 by 23000. *Ans.* 2005600000.
15. Multiply 90700 by 50700. *Ans.* 4598490000.
16. Multiply 876000 by 25600. *Ans.* 22425600000.
17. Multiply 807000 by 29000. *Ans.* 23403000000.
18. Multiply 908000 by 327000. *Ans.* 296916000000.
19. Multiply 2870000 by 108000. *Ans.* 309960000000.
20. Multiply 607000 by 9080000. *Ans.* 5511560000000.

WRITTEN EXERCISES.

IN ADDITION, SUBTRACTION, AND MULTIPLICATION.

1. I bought 3 spades at $1.50 apiece, 5 shovels at $1.25 apiece, and 4 rakes at $0.75 apiece; what was the entire bill? *Ans.* $13.75.

2. A's income is $5875 a year; he pays $975 for house-rent, and 3 times as much for other expenses; how much can he save annually? *Ans.* $1975.

3. An army of 24,500 men lost in a battle 246 killed, 4 times as many, plus 145, wounded, and 1687 captured; how many remained in the army? *Ans.* 21,438 men.

4. Mary and Martha tried which could count the greater number in 25 minutes; Mary averaged 76 a minute, and Martha 81 a minute; how many did Martha count more than Mary? *Ans.* 125.

5. A poor boy attempted to walk from Philadelphia to Pittsburgh, a distance of 353 miles; after walking 18 miles a day for 15 days, how far was he from Pittsburgh?

Ans. 83 miles.

6. The light from a certain star is 7 years in coming to the earth; what is its distance, supposing there are 86,400 sec-

onds in one day, and 365 days in a year, light moving at the rate of 192,500 miles a second? *Ans.* 42494760 millions.

7. A wholesale druggist bought as follows: 500 lb. opium at $7.20 per lb.; 375 lb. morphine at $4.75 per lb.; 240 lb. quinine, at $2.70 per lb.; 200 lb. camphor, at $0.29 per lb.; what was his whole bill? *Ans.* $6087.25.

8. A clerk's salary is $1500 a year; he pays $5 a week for his board and washing, $4 a month for car fare, and his other expenses amount to about $2 a day; how much can he save in a year? *Ans.* $462.

9. Mrs. Wilson bought at a furniture store an oiled walnut chamber set for $150, two cottage sets for $55 each, a dozen dining-room chairs at $3.50 each, and 3 camp chairs at $4.25 each; what was her bill? *Ans.* $314.75.

10. A shipping firm bought 150 lb. Manila bolt rope yarns at 16 cents per lb.; 450 lb. Manila whale lines at 17 cents a pound; 260 lb. Russia hemp tarred cordage, at 13 cents per lb.; and 850 lb. Sisal rope at 11 cents per lb.; what was the whole cost? *Ans.* $227.80.

11. A dry goods merchant sold to a Western customer as follows; 125 yards Cocheco fancy prints, at 7 cents a yard; 147 yards Cocheco robes, 8 cents; 156 yards Merrimac robes, 8 cents; 122 yards Merrimac checks and stripes, 7 cents; 125 yards French cashmeres, $1.15; what was the amount of the bill? *Ans.* $185.28.

12. James Chauncey & Co. sold to Barton Brothers, Dec. 23, 1875, the following: 450 bushels White Western oats, at 38 cents a bushel; 375 bushels New York No. 2 White oats, 43 cents; 240 bushels choice new Southern Black-eye peas, $1.45. Jan. 10, 1876, J. C. & Co. bought of Barton Brothers, 460 bushels Pennsylvania rye, 91 cents; 175 bushels No. 2 Canada barley, $1.10; 720 bushels common Winter Red Western wheat, $1.25; required the balance of the account. *Ans.* $830.85 favor of Barton Brothers.

INTRODUCTION TO DIVISION.

MENTAL EXERCISES.

1. At 3 cents each, how many melons can I buy for 15 cents?

SOLUTION.—If 1 melon costs 3 cents, for 15 cents I can buy as many melons as 3 is contained times in 15, which are 5.

2. How many yards of ribbon, at 8 cents a yard, can be bought for 56 cents?

3. A man gave 60 dollars for sheep, at the rate of 5 dollars a head how many did he buy?

4. How many kegs, of 9 gallons each, can be filled from a hogshead containing 90 gallons of vinegar?

5. How many days must a man work, at the rate of $3 a day, to earn $45?

6. How many lemons at 6 cents apiece may be bought for 84 cents?

7. How many are 15 plus 5, divided by 5? 18 plus 6, divided by 6? 40 plus 8, divided by 8? 35 plus 7, divided by 7?

8. How many are 3 times 8 divided by 4? 5 times 9 divided by 3? 6 times 10 divided by 12? 8 times 7 divided by 4?

9. How many are 3 times 33 divided by 11? 4 times 21 divided by 7? 3 times 25 divided by 5? 3 times 24 divided by 8?

10. How many pencils, worth 5 cents each, may be bought for 4 erasers worth 15 cents each?

11. How many barrels of flour, at $9 a barrel, can be bought for 15 barrels of apples at $3 a barrel?

12. A woman carried to the store 6 dozen eggs at 25 cents a dozen; how many yards of calico, at 10 cents a yard, will pay her?

13. If mackerel is worth $12 a barrel, how many barrels can be bought for $6 in money and 6 barrels of pork at $15 a barrel?

14. The process of finding how often one number is contained in another is called *division*. The result is called the *quotient*.

15. The number divided is the *dividend;* the number we divide by is the *divisor*. The sign of division is ÷, and is read, "divided by."

16. The sign ( ) signifies that the quantities included are to be subjected to the same operation.

17. Find the result of the following:

(13+5)÷3	(25—5)÷4	(6× 6)÷ 4	32÷(2×4)
(14+6)÷4	(26—2)÷3	(8× 9)÷12	36÷(3×4)
(15+7)÷2	(28—3)÷5	(5×12)÷10	42÷(2×7)
(17+4)÷7	(32—2)÷6	(4×12)÷ 8	48÷(3×2)
(18+6)÷8	(34—6)÷7	(7× 9)÷ 3	54÷(3×3)

DIVISION.

82. Division is the process of finding the *quotient* of **two** numbers.

83. The **Quotient** of two numbers is a number which expresses how often one number is contained in another.

84. The **Dividend** is the number to be divided.

85. The **Divisor** is the number by which we divide.

86. The **Remainder** is the number which is sometimes left after dividing.

87. The **Sign of Division** is ÷, and is read *divided by.* It denotes that the number preceding it is to be divided by the number following it.

NOTES.—1. The sign of division is a short line, in the line of writing, with one dot above and another below the middle of it.

2. The symbol ÷ was introduced by Dr. John Pell, an English mathematician, born in 1610.

3. Division is also indicated by writing the divisor beneath the dividend, with a straight line between them; or by writing the divisor at the left of the dividend, with a curved line between them; thus $\frac{27}{9}$, and 9)27, mean 27 divided by 9.

PRINCIPLES.

1. *The divisor and dividend are similar numbers.*

2. *The quotient is an abstract number; the remainder is similar to the dividend.*

3. *In dividing a number into equal parts, the dividend and quotient are similar, and the divisor is abstract.*

METHODS OF DIVISION.

88. There are **Two Methods** of performing division, called *Short Division* and *Long Division.*

SHORT DIVISION.

89. Short Division is that method in which only the dividend, divisor, and quotient are written, the operation being performed mentally.

90. Short division is generally employed when the divisor does not exceed *twelve*, the largest multiplier in the multiplication table.

1. How many times is 2 contained in 358?

SOLUTION.—We write the divisor at the left of the dividend, with a curved line between them, draw a line beneath the dividend, and begin at the left to divide. 2 is contained in 3 hundreds 1 hundred times, and 1 hundred remaining; 1 hundred equals 10 tens, which with 5 tens are 15 tens: 2 is contained in 15 tens 7 tens times, with a remainder of 1 ten; 1 ten equals 10 units, which with 8 units equals 18 units: 2 is contained in 18 units 9 units times, and we have for the quotient, 179. Hence we have the following

OPERATION.

2)358

179

Rule.—I. *Write the divisor at the left of the dividend, with a curved line between them and a line beneath the dividend.*

II. *Begin at the left, divide each term of the dividend by the divisor, and write the quotient beneath.*

III. *If there is a remainder after any division, regard it as prefixed to the next term of the dividend, and divide as before.*

IV. *If any partial dividend is less than the divisor, write a cipher in the quotient and prefix the dividend to the next term.*

V. *When there is a final remainder, annex it, with the divisor written beneath, to the integral part of the quotient.*

Proof.—Multiply the quotient by the divisor, and add the remainder, if any, to the product; if the work is correct, the result will equal the dividend.

NOTE.—In practice we need not name the denomination of the different partial dividends. Thus, in the above solution we say 2 is contained in 3, once and one remaining; 2 is contained in 15, 7 times, etc.

WRITTEN EXERCISES.

(2)	(3)	(4)	(5)	(6)
2)372	2)456	3)618	3)852	3)783
186	228	206	284	261
(7)	**(8)**	**(9)**	**(10)**	**(11)**
4)968	4)556	4)988	5)850	5)960
242	139	247	170	192

12. Divide 1395 by 5. *Ans.* 279.

13. Divide 6906 by 6. *Ans.* 1151.

14. Divide 3248 by 7. *Ans.* 464.

15. Divide 7185 by 7. *Ans.* 1026$\frac{3}{7}$.

16. Divide 8097 by 7. *Ans.* 1156$\frac{5}{7}$.

17. Divide 9136 by 8. *Ans.* 1142.

18. Divide 72352 by 8. *Ans.* 9044.

19. Divide 91672 by 8. *Ans.* 11459.

20. Divide 23769 by 9. *Ans.* 2641.

21. Divide 30564 by 9. *Ans.* 3396.

22. Divide 9876504 by 7. *Ans.* 1410929$\frac{1}{7}$.

23. Divide 3201567 by 6. *Ans.* 533594$\frac{3}{6}$.

24. Divide 4187002 by 8. *Ans.* 523375$\frac{2}{8}$.

25. Divide 83702507 by 9. *Ans.* 9300278$\frac{5}{9}$.

26. If a book cost 4 dollars, how many books, at the same rate, can you buy for 252 dollars?

SOLUTION.—If 1 book cost 4 dollars, for 252 dollars we can buy as many books as 4 dollars are contained times in 252 dollars, which are 63. Therefore, etc.

OPERATION.

4)252

63

27. There are 3 feet in one yard; how many yards in 291 feet? *Ans.* 97 yards.

28. There are 8 quarts in one peck; how many pecks are there in 1728 quarts? *Ans.* 216 pecks.

29. There are 7 days in one week; how many weeks in 364 days? *Ans.* 52 weeks.

30. A man gave 324 dollars to some boys, giving 6 dollars to each; how many boys were there? *Ans.* 54.

31. If one sheep cost 9 dollars, how many sheep can you buy for 1935 dollars? *Ans.* 215 sheep.

32. If there are 12 pence in 1 shilling, how many shillings are there in 571,836 pence? *Ans.* 47,653 shillings.

33. If it require one sheet of paper to print 12 pages of a book, how many sheets will be required for a book of 504 pages? *Ans.* 42 sheets.

34. How long will it take two boys, starting at the same place, and traveling in opposite directions, to be 29,076 rods apart, if one goes 5 and the other 7 rods in a minute?

Ans. 2423 minutes.

LONG DIVISION.

91. Long Division is the method of dividing when the work is written out in full. It is generally used when the divisor exceeds 12.

1. Divide 5848 by 23.

SOLUTION.—23 is not contained in 5 thousands any thousands times, hence there are no thousands in the quotient. 5 thousands and 8 hundreds are 58 hundreds; 23 is contained in 58 hundreds 2 hundreds times: 2 hundreds times 23 are 46 hundreds, which subtracted from 58 hundreds leave 12 hundreds: 12 hundreds and 4 tens are 124 tens; 23 is contained in 124 tens 5 tens times: 5 tens times 23 are 115 tens, which subtracted from 124 tens leave 9 tens: 9 tens and 8 units are 98 units; 23 is contained in 98 units 4 times; 4 times 23 equals 92: subtracting there is a remainder of 6, which will not contain 23; hence the quotient is 2 hundreds, 5 tens, and 4 units, or 254, with a remainder of 6.

OPERATION.

```
23)5848(254
   46
  ---
  124
  115
  ---
   98
   92
  ---
    6
```

Rule.—I. *Draw curved lines at both sides of the dividend, and place the divisor at the left.*

II. *Divide the number expressed by the fewest terms at the left that will contain the divisor, and place the quotient at the right.*

III. *Multiply the divisor by this quotient, write the product under the partial dividend, subtract, and to the remainder annex the next term of the dividend.*

IV. *Divide as before, and thus continue until all the terms of the dividend have been used.*

V. *If any partial dividend will not contain the divisor, place a cipher in the quotient, annex the next term of the dividend, and proceed as before.*

VI. *When there is a final remainder, annex it, with the divisor written beneath, to the integral part of the quotient.*

Proof.—Multiply the integral part of the quotient by the divisor, and add the remainder, if any, to the product; if the work is correct the result will be equal to the dividend.

NOTES.—I. The pupils will notice that there are *five* operations: 1st. *Write the number;* 2d. *Divide;* 3d. *Multiply;* 4th. *Subtract;* 5th. *Bring down*

II. Pupils often have difficulty in finding the correct quotient figure: this difficulty can be greatly diminished by attention to the following suggestions:

1st. Notice how often the left-hand term of the divisor is contained in the term or terms of the partial dividend, as far from the right hand term as the left hand term in the divisor is from the right hand term.

2d. If, when we multiply, the product is greater than the partial dividend, the quotient term must be diminished.

3d. If, when we subtract, the remainder is greater than the divisor, the quotient term must be increased.

III. We commence at the left to divide, so that the remainder can be united to the number of units of the next lower order, giving a new partial dividend. The sign + is used to denote a remainder.

WRITTEN EXERCISES.

1. Divide 840 by 24.	*Ans.* 35.
2. Divide 903 by 21.	*Ans.* 43.
3. Divide 455 by 13.	*Ans.* 35.
4. Divide 4956 by 14.	*Ans.* 354.
5. Divide 3510 by 15.	*Ans.* 234.
6. Divide 5024 by 16.	*Ans.* 314.
7. Divide 4720 by 20.	*Ans.* 236.
8. Divide 5778 by 18.	*Ans.* 321.
9. Divide 8736 by 21.	*Ans.* 416.
10. Divide 9591 by 23.	*Ans.* 417.
11. Divide 4056 by 24.	*Ans.* 169.
12. Divide 5450 by 25.	*Ans.* 218.
13. Divide 9672 by 26.	*Ans.* 372.
14. Divide 7946 by 29.	*Ans.* 274.
15. Divide 9840 by 30.	*Ans.* 328.
16. Divide 11286 by 27.	*Ans.* 418.
17. Divide 17500 by 28.	*Ans.* 625.
18. Divide 13872 by 34.	*Ans.* 408.
19. Divide 24588 by 36.	*Ans.* 683.
20. Divide 25536 by 38.	*Ans.* 672.
21. Divide 31629 by 39.	*Ans.* 811.
22. Divide 29725 by 41.	*Ans.* 725.
23. Divide 28896 by 43.	*Ans.* 672.
24. Divide 72945 by 45.	*Ans.* 1621.
25. Divide 81098 by 46.	*Ans.* 1763.
26. Divide 60466 by 49.	*Ans.* 1234.
27. Divide 141050 by 50.	*Ans.* 2821.
28. Divide 316160 by 52.	*Ans.* 6080.
29. Divide 333234 by 54.	*Ans* 6171.

30. Divide 105056 by 56. *Ans.* 1876.
31. Divide 126498 by 58 *Ans.* 2181.
32. Divide 172929 by 59. *Ans.* 2931.
33. Divide 173911 by 61. *Ans.* 2851.
34. Divide 178857 by 63. *Ans.* 2839.
35. Divide 1568580 by 65 *Ans.* 24132.
36. Divide 1380536 by 68. *Ans.* 20302.
37. Divide 692208 by 69. *Ans.* 10032.
38. Divide 2434380 by 78. *Ans.* 31210.
39. Divide 1031475 by 85. *Ans.* 12135.
40 Divide 1680137 by 97. *Ans.* 17321.
41. Divide 2138654 by 98. *Ans.* 21823.
42. Divide 317646 by 126. *Ans.* 2521.
43. Divide 238788 by 134. *Ans.* 1782.
44. Divide 456104 by 146. *Ans.* 3124.
45. Divide 603264 by 192. *Ans.* 3142.
46. Divide 711287 by 321. *Rem.* 272.
47. Divide 811332 by 372. *Ans.* 2181.
48. Divide 1646301 by 381. *Ans.* 4321.
49. Divide 1985175 by 425. *Ans.* 4671.
50. Divide 1957413 by 453 *Ans.* 4321.
51. Divide 1787160 by 562. *Ans.* 3180.
52. Divide 2100315 by 581. *Ans.* 3615.
53. Divide 3207594 by 767. *Ans.* 4182.
54. Divide 1019806 by 893. *Ans.* 1142.
55. Divide 7481888 by 1021. *Ans.* 7328.
56. Divide 5226412 by 2036. *Ans.* 2567.
57. Divide 6419945 by 2135 *Ans.* 3007.
58. Divide 13824902 by 3367. *Ans.* 4106.
59. Divide 8074528 by 6328. *Ans.* 1276.
60. Divide 97547337 by 3891. *Rem.* 3858.
61. Divide 4223745376 by 23456. *Ans.* 180071.
62. Divide 170627676887 by 413071. *Rem.* 25846.
63. Divide 129652565329 by 360073. *Ans.* 360073.
64. Multiply 37602 by 608, and divide the product by 304. *Ans.* 75204.

PROBLEMS IN DIVISION.

92. In **Division** there are two classes of practical problems:

1st. To find the number of equal parts of a number.

2d To divide a number into equal parts.

CASE I.

93. *To find the number of equal parts of a number.*

MENTAL EXERCISES.

1. At 8 cents a yard how many yards of ribbon can I buy for 72 cents?

SOLUTION.—If 1 yard of ribbon costs 8 cents, for 72 cents I can buy as many yards as 8 is contained times in 72, which are 9.

2. How many oranges can I buy for 48 cents, at the rate of 8 cents apiece?

3. A farmer expended $96 in buying sheep at $8 apiece; how many sheep did he buy?

4. How many melons at 7 cents apiece may be bought for 84 cents?

5. How many oranges at 5 cents apiece may be bought for 7 lemons worth 10 cents apiece?

6. How many yards of lace at 8 cents a yard may be bought for 6 yards of muslin at 12 cents a yard?

7. How much wheat worth 6 dimes a bushel may be bought for 12 bushels of corn at 5 dimes a bushel?

8. If flour is worth $8 a barrel, how many barrels could be exchanged for $12 in money and 4 barrels of fish worth $12 a barrel?

9. A man gave 7 pencils, worth 5 cents each, for 4 packages of envelopes, worth 10 cents each; what was the gain?

10. Divide 96 by 4.

SOLUTION.—4 is contained in 9 tens 2 tens times and 1 ten remaining; 1 ten and 6 units equal 16 units; 4 is contained in 16 units 4 units times; hence the quotient is 2 tens and 4 units, or 24.

11. Divide 68 by 2; 69 by 3; 56 by 4; 72 by 3; 75 by 5; 76 by 4; 126 by 6; 147 by 7; 176 by 8.

12. Divide 720 by 8; 350 by 7; 720 by 9, 156 by 12; 231 by 11; 1224 by 12; 2408 by 8; 2718 by 9; 1728 by 12.

13. If you earn $15 a week and pay $5 for board and $4 for other expenses, in what time could you save $72?

14. Two men start from places 180 miles apart, and travel towards each other, one going 4 and the other 5 miles an hour; in how many hours will they meet?

WRITTEN EXERCISES.

1. At 25 dollars each, how many cows can be bought for 575 dollars?

SOLUTION.—If 25 dollars will buy one cow, 575 dollars will buy as many cows as 25 dollars are contained times in 575 dollars, which are 23.

OPERATION.

25)575(23

2. In one hogshead there are 63 gallons; how many hogsheads in 15,435 gallons? *Ans.* 245 hhd.

3. How many horses can you get for 1824 dollars, at the rate of 152 dollars each? *Ans.* 12 horses.

4. If a boat sails 25 miles an hour, how long will it be in sailing 1800 miles? *Ans.* 72 hours.

5. How many years must a person labor to earn $13,140, at the rate of $730 a year? *Ans.* 18 years.

6. How many acres of land can you purchase for $11,696, at the rate of $86 an acre? *Ans.* 136.

7. How many cows, at $37 each, can be bought for 74 horses, at $150 each? *Ans.* 300 cows.

8. How many oxen, at $54 each, can be bought for 108 mules, at $94 each? *Ans.* 188 oxen.

9. A labors 72 weeks, at $14 a week; how much wheat at 42 cents a bushel will pay him? *Ans.* 2400 bushels.

10. The circumference of the earth is 25,000 miles; how long would it take a vessel to sail around it, going at the rate of 125 miles per day? *Ans.* 200 days.

11. The distance from the earth to the sun is 93,000,000 miles; how long will it take light to reach us from the sun, moving 11,520,000 miles a minute? *Ans.* 8 min.+.

12. The moon is 240,000 miles from the earth; how long would it take a balloon to reach it, moving at the rate of 75 miles an hour? *Ans.* 3200 hours.

13. A builder received Western lands at $25 per acre and a balance in cash of $7300, in trade for a row of 15 houses at $2575 each; how many acres did he receive? *Ans.* 1253.

14. A person wishes to trade land worth 150 dollars an acre, for 125 acres, at 75 dollars an acre, and gain 675 dollars by the bargain; how many acres will be exchanged?

Ans. 58.

CASE II.

94. *To divide a number into equal parts.*

MENTAL EXERCISES.

1 What is one of the 4 equal parts of 24?

SOLUTION.—One of the 4 equal parts of 24 is 6, since 6 taken 4 times is 24.

2. Find one of the 4 equal parts of 12; of 20; of 36; of 48; of 32; of 40; of 44.

3. Find one of the 5 equal parts of 15; of 25; of 30; one of the 6 equal parts of 42; of 48; of 60; of 72.

4. Find one of the 7 equal parts of 35; of 56; of 63; one of the 8 equal parts of 40; of 72; of 88; of 96.

5. What is one-third of anything? *Ans.* One third of anything is one of the 3 equal parts into which it can be divided.

6. What is one-half of anything? one-fourth? one-fifth? one-sixth? one-seventh? one-eighth? one-ninth? one-tenth?

7. What is one-fourth of 24 dollars?

SOLUTION.—One-fourth of $24 is $6, since $6 taken 4 times equals $24.

8. What is one-fourth of 20? of 36? of 48? of 56? of 72?

9. What is one-third of 21? of 27? of 30? of 36? of 48?

10. What is one-fifth of 15? of 30? of 40? of 45? of 60?

11. What is one-sixth of 48? of 72? of 84? of 90? of 96?

12. If 6 slates cost $1.20, how much is that apiece?

SOLUTION.—If 6 slates cost $1.20, one slate cost one-sixth of $1.20, which is 20 cents.

13. If 8 pairs of gloves cost $2.40, how much is that a pair?

14. If 6 cords of wood cost $48, what is the price per cord?

15. What do I pay a bushel for apples if 12 bushels cost me $4.80?

16. What does a lady pay a yard for lace if 12 yards cost her $7.20?

WRITTEN EXERCISES.

1. Divide 235 into 5 equal parts.

SOLUTION 1ST.—If we divide 235 into 5 equal parts, each part is $\frac{1}{5}$ of 235: $\frac{1}{5}$ of 23 tens is 4 tens and 3 tens remaining; 3 tens and 5 units equal 35; $\frac{1}{5}$ of 35 is 7; hence $\frac{1}{5}$ of 235 is 47, or 47 is one of the 5 equal parts of 235.

OPERATION.

$$5)\overline{235}\\ \overline{47}$$

SOLUTION 2D.—One of the five equal parts of 5 is *one*, hence one of the 5 equal parts of 235 is as many times *one* as 5 is contained times in 235, which are 47. Therefore, etc.

2. Divide 212 into 4 equal parts. *Ans.* 53.

3. Divide 222 into 6 equal parts. *Ans.* 37.

4. Divide 455 into 7 equal parts. *Ans.* 65
5. Divide 592 into 8 equal parts. *Ans.* 74.
6. Divide 425 into 17 equal parts. *Ans.* 25
7. Divide 608 into 19 equal parts. *Ans.* 32.
8. Divide 1107 into 9 equal parts. *Ans.* 123.
9. Divide 2574 into 11 equal parts. *Ans.* 234.
10. Divide 3780 into 12 equal parts. *Ans.* 315.
11. Divide 3168 into 24 equal parts. *Ans.* 132.
12. Divide 10725 into 33 equal parts. *Ans.* 325.
13. Divide 205896 into 46 equal parts. *Ans.* 4476.
14. A man divides $318 among 6 boys; how much will each one receive?

SOLUTION 1ST.—It will require $6 to give each boy $1; and in giving $318, each boy will receive as many dollars as $6 are contained times in $318, which are 53. Therefore, etc.

OPERATION.

$$6)318$$
$$53$$

SOLUTION 2D.—If 6 boys receive $318, one boy will receive one-sixth of $318, which, by division, we find is $53. Therefore, etc.

15. If 12 men earn $384 in a week, how much does one man earn? *Ans.* $32.
16. A boat goes 1584 miles in 24 hours; how far will it go in 1 hour? *Ans.* 66 miles.
17. There are 1575 gallons in 25 hogsheads; how many gallons are there in 1 hogshead? *Ans.* 63 gallons.
18. There are 8316 cubic inches in 36 gallons of wine; how many cubic inches are there in 1 gallon? *Ans.* 231.
19. There are 7614 cubic inches in 27 gallons of beer; how many cubic inches in one gallon? *Ans.* 282.
20. There are 221,760 feet in 42 miles; how many feet are there in one mile? *Ans.* 5280.
21. Sound moves 61,545 feet in 55 seconds; how far does it move in one second? *Ans.* 1119 feet.
22. If a turnpike 132 miles long cost $339,240, how much did it cost per mile? *Ans.* $2570.
23. The salary of the President of the United States is $50,000 a year; what is it a day? *Ans.* $137 nearly.
24. A man having $20.000 buys 150 acres of land, at $75 an acre; how much land can he buy with what remains, at $125 an acre? *Ans.* 70 acres.

CONTRACTIONS IN DIVISION.

95. Contractions in Division are abbreviated forms of dividing.

CASE I.

96. ***When the divisor is a composite number.***

1. Divide 2952 by 24, using the factors 4 and 6.

SOLUTION 1ST.—To *multiply* by 24 we may multiply by 6, and then multiply that product by 4; hence, to *divide* by 24 we may divide by 4, and then divide that quotient by 6. Dividing by 4 we have 738, and dividing 738 by 6 we have 123; hence, etc.

OPERATION.

4)2952
6)738
123

SOLUTION 2D.—Since 24 times a number equals 6 times 4 times the number, $\frac{1}{24}$ of the number equals $\frac{1}{6}$ of $\frac{1}{4}$ of the number; $\frac{1}{4}$ of 2952 is 738, and $\frac{1}{6}$ of 738 is 123; hence, etc.

Rule.—*Divide the dividend by one factor of the divisor, the quotient by another factor, and thus continue for all the factors used; the last quotient will be the quotient required.*

WRITTEN EXERCISES.

Divide the following, using the factors:

2. 570 by 15.	(15=3×5)	*Ans.* 38.
3. 492 by 12.	(12=4×3)	*Ans.* 41.
4. 594 by 18.	(18=3×6)	*Ans.* 33.
5. 2480 by 20.	(20=4×5)	*Ans.* 124.
6. 4494 by 14.	(14=2×7)	*Ans.* 321.
7. 10950 by 30.	(30=5×6)	*Ans.* 365.
8. 7875 by 35.		*Ans.* 225.
9. 12560 by 40.		*Ans.* 314.
10. 22824 by 72.		*Ans.* 317.
11. 47412 by 108.		*Ans.* 439.
12. 64440 by 120.		*Ans.* 537.
13. 54576 by 144.		*Ans.* 379

TO FIND THE TRUE REMAINDER.

97. The **True Remainder** in successive division, it is evident, is not the last remainder, nor the sum of all the remainders; it is necessary, therefore, to explain the method of finding the true remainder.

1. Divide 791 by 24, using the factors 2, 3, and 4.

SOLUTION.—Dividing by 2 we find that 791 equals 395 *twos* and 1 remaining; dividing 395 *twos* by 3, we find 395 *twos* equals 131 *sixes* and 2 *twos*, or 4, remaining; dividing by 4, we find that 131 *sixes* consists of 32 *twenty-fours* and 3 *sixes*, or 18, remaining. Hence the true remainder is 18+4+1, which is 23. Hence, to find the correct remainder we have the following

OPERATION.

2)791
3)395 1
4)131, 2 *twos* = 4
32, 3 *sixes* = 18
True remainder, 23.

Rule.—*Multiply each remainder by all the divisors preceding the one which obtained it, and take the sum of the products and the remainder arising from the first division*

Divide the following and find the true remainder:

2. 582 by 15.		*Rem.* 12.
3. 503 by 12.		*Rem.* 11.
4. 2497 by 20.		*Rem.* 17.
5. 4507 by 14.		*Rem.* 13.
6. 3717 by 30.	2, 3, 5.	*Rem.* 27.
7. 13853 by 105.	3, 5, 7.	*Rem.* 98.
8. 41837 by 180.	4, 5, 9.	*Rem.* 77.
9. 47117 by 308.	4, 7, 11.	*Rem.* 301.
10. 96711 by 310.	2, 5, 31.	*Rem.* 301.
11. 87831 by 720.	2, 3, 4, 5, 6.	*Rem.* 711.

CASE II.

98. ***When there are ciphers at the right of the divisor.***

1. Divide 8254 by 600.

SOLUTION.—6 *hundreds* are contained in 82 *hundreds* 13 times, and 400 remaining; 600 is not contained in 54, hence the entire remainder is 400+54, or 454. From this solution we may derive the following

OPERATION.

6|00)82|54
13–454

Rule.—I. *Cut off the ciphers at the right of the divisor and as many terms at the right of the dividend.*

II. *Divide the remaining part of the dividend by the remaining part of the divisor.*

III. *Prefix the remainder to the part of the dividend cut off, and the result will be the true remainder.*

NOTES.—1. When the divisor is a unit of any order with ciphers, the remainder will be the figures cut off at the right, and the quotient the figures at the left.

2. When the part of the divisor at the left of the naughts is greater than 12, divide by long division.

2. Divide 876 by 50. *Ans.* 17; *Rem.* 26.
3. Divide 953 by 400. *Ans.* 2; *Rem.* 153.
4. Divide 1733 by 500. *Ans.* 3; *Rem.* 233.
5. Divide 2765 by 700. *Ans.* 3; *Rem.* 665.
6. Divide 7859 by 800. *Ans.* 9; *Rem.* 659.
7. Divide 9763 by 900. *Ans.* 10; *Rem.* 763.
8. Divide 14873 by 1900. *Ans.* 7; *Rem.* 1573.
9. Divide 25075 by 2300. *Ans.* 10; *Rem.* 2075.
10. Divide 187654 by 14700. *Rem.* 11254.
11. Divide 269856 by 237000. *Rem.* 32856.
12. Divide 5767220 by 4730000. *Rem.* 1037220.

EXERCISE UPON THE PARENTHESIS.

99. The **Parenthesis** ( ), denotes that the quantities included are to be subjected to the same operation; thus, $(8+6-4)\times 3$ denotes that the value of $8+6-4$, which is 10, is to be multiplied by 3.

The *vinculum*, thus $\overline{8+6-4}\times 3$, is often used in place of the parenthesis.

1. What is the value of $(12+9-7)\times 5$?

SOLUTION.—12+9 equals 21, and 21 minus 7 equals 14, and 14 multiplied by 5 equals 70. Therefore, etc.

Required the value

2. Of $(25+23-18)\times 7$. *Ans.* 210.
3. Of $(46+97-82)\times 9$. *Ans.* 549.
4. Of $(98-75+87)\times 14$. *Ans.* 1540.
5. Of $(89+96-47)\div 6$. *Ans.* 23.
6. Of $(145-110+117)\div 8$. *Ans.* 19.
7. Of $(396-128+483)\times 32$. *Ans.* 24032.
8. Of $(860+980-1120)\div 45$. *Ans.* 16.
9. Of $(320-98)\times(860-145)$. *Ans.* 158730.
10. Of $(689-327+986-397)\times 428$. *Ans.* 407028.
11. Of $(729+487-244)\div(247-210+71)$. *Ans.* 9.
12. Of $(3014-2601)\times(2477-1325)\div 59$. *Ans.* 8064.

NOTE.—In a series of numbers connected with symbols, the sign $\times$ denotes the closest connection, the sign $\div$ next, thus, $12+8\div 2-5\times 2=6$; also, $16\div 4\times 2=2$, rather than 8.

WRITTEN EXERCISES.

ON THE FOUR FUNDAMENTAL RULES.

1. The minuend is 4160, and the subtrahend is 3425; what is the remainder? *Ans.* 735.

2. The minuend is 9164 and the remainder is 3426; what is the subtrahend? *Ans.* 5738.

3. The subtrahend is 3872 and the remainder 4648; what is the minuend? *Ans.* 8520.

4. The multiplicand is 745 and the multiplier 456; what is the product? *Ans.* 339720.

5. The multiplicand is 2463 and the product 854661; what is the multiplier? *Ans.* 347.

6. The product is 881919 and the multiplier 981; what is the multiplicand? *Ans.* 899.

7. The dividend is 518077 and the divisor 763; what is the quotient? *Ans.* 679.

8. The dividend is 801222 and the quotient 3257; what is the divisor? *Ans.* 246.

9. The divisor is 587 and the quotient 8723; what is the dividend? *Ans.* 5120401.

10. The dividend is 72987 and divisor 45; required the quotient and remainder. *Ans.* 1621; 42.

11. The dividend is 7972, the quotient is 274, and remainder 26; what is the divisor? *Ans.* 29.

12. The divisor is 26, the quotient 372, and remainder 23; what is the dividend? *Ans.* 9695.

13. Thomas read 789 pages of history in a week, which lacks 324 of being as many as Walton read; how many did both read? *Ans.* 1902 pages.

14. A freight car ran 365 miles one week, and 3 times as far, lacking 246 miles, the next week; how far did it run the second week? *Ans.* 849 miles.

15. A sold 8318 bushels of wheat, then bought 2514 bushels, and then had 3146 bushels; how many bushels had he at first? *Ans.* 8950 bushels.

16. My barn cost $3156; my house cost 3 times as much as my barn, and my farm cost as much as both; what was the cost of all? *Ans.* $25,248.

17. The value of 5 horses and 7 mules is $2436; now if the value of each mule is $208, what is the value of each horse? *Ans.* $196.

18. A man left $2535 each to his four children, but one of them dying, the three remaining children divided the money; how much did each receive? *Ans.* $3380.

19. Mr. Smith left $6264 to each of three sons and $7240 to each of two daughters, but one daughter dying, her share was equally divided among the remaining children; what did each receive? *Ans.* Son, $8074; daughter, $9050.

20. The income of a man who "struck oil" was $480 a day; how many teachers would this employ at a salary of $438 a year? *Ans.* 400.

21. A stock dealer bought 325 cows at $28 each, and sold 124 of them at cost; how much must he receive a head for the remainder to gain $804? *Ans.* $32.

22. Mr. Galton buys a farm of 110 acres at $75 an acre, $2200 to be paid down and the remainder in five yearly installments; what must he pay each year? *Ans.* $1210.

23. A farmer raised 765 bushels of oats, of which he kept 65 bushels for seed, and after retaining enough for the use of his horses till next harvest, allowing 60 bushels to each horse, sold the balance at 85 cents a bushel, and received $442; how many horses had he? *Ans.* 3 horses.

24. Mr. Milman bequeathed $6500 to each of two sons, to a third son $1000, $5000 to each of 3 daughters, and the balance of his estate, amounting to $25,000, to several benevolent institutions; the will, however, being set aside, the property was divided equally among his children; what was the share of each? *Ans.* $9000.

25. If a soldier enlisting in the late war for 3 years, received a bounty of $930; then served one year as a private, at $13 a month, 6 months as a corporal, at $14 a month, and 18 months as a sergeant at $17 a month; what was the whole amount of his pay and his average pay per month?

Ans. $41 a month.

GENERAL PRINCIPLES

OF THE FUNDAMENTAL RULES.

PRINCIPLES OF ADDITION.

1. The sum of all the parts equals the whole.
2. The whole diminished by one or more parts equals the sum of the other parts.

PRINCIPLES OF SUBTRACTION.

1. The Remainder equals the Minuend minus the Subtrahend.
2. The Minuend equals the Subtrahend plus the Remainder.
3. The Subtrahend equals the Minuend minus the Remainder.

PRINCIPLES OF MULTIPLICATION.

1. The Product equals the Multiplicand into the Multiplier.
2. The Multiplicand equals the Product divided by the Multiplier.
3. The Multiplier equals the Product divided by the Multiplicand.

PRINCIPLES OF DIVISION.

1. The Quotient equals the Dividend divided by the Divisor.
2. The Dividend equals the Divisor multiplied by the Quotient.
3. The Divisor equals the Dividend divided by the Quotient.
4. The Dividend equals the Divisor multiplied by the Quotient plus the Remainder.
5. The Divisor equals the Dividend minus the Remainder, divided by the Quotient.

OTHER PRINCIPLES OF DIVISION.

1. Multiplying the Dividend or dividing the Divisor by any number, multiplies the Quotient by that number.
2. Dividing the Dividend or multiplying the Divisor by any number, divides the Quotient by that number.
3. Multiplying or dividing both Dividend and Divisor by the same number, does not change the Quotient.
4. *A General Law.*—A change in the Dividend by multiplication or division produces a *similar* change in the Quotient; but such a change in the Divisor produces an *opposite* change in the Quotient.

NOTE TO TEACHER.—Let the pupils be required to show the reason for the above principles, and give illustrations of them. No demonstrations are given, since it is better for the pupil to learn to depend somewhat upon himself, that he may become, not a mere imitator, but an *original thinker*.

INTRODUCTION TO SECONDARY OPERATIONS.

MENTAL EXERCISES.

1. What numbers multiplied together will produce 4? 6? 8? 10? 12? 14? 16? 18? 20? 24? 26? 28? 30?

2. What numbers can be produced out of the numbers 2 and 3? 3 and 5? 2, 3, and 5? 3, 4, and 5? 2, 3, 4, and 5?

3. Will the product of any two numbers, each greater than a unit produce 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, and 37?

4. What may we call a number which is *composed* by multiplying several numbers together? *Ans.* A *Composite Number.*

5. What shall we call numbers that cannot be produced by multiplying several numbers together? *Ans. Prime Numbers.*

6. Which are prime and which composite numbers in the following list: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15?

7. What may we call the numbers whose product makes a composite number? *Ans. Makers* of the numbers.

8. If the word *Factor* means the same as *maker*, what may we call the *makers* of a composite number? *Ans. Factors.*

9. Form composite numbers out of the factors 3 and 4; 3, 4, and 6; 4, 5, and 6. What are the factors of 12? 15? 18? 20? 21? 24?

10. Form a composite number by using 2 twice as a factor; 3 twice as a factor; 2 three times as a factor; 3 four times as a factor.

11. Required *one* of the *two equal* factors of 9; of 16; of 25; of 36 *one* of the *three equal* factors of 8; of 27; of 64; of 125.

12. A number composed of two equal factors is called the *second power* of that factor; of three equal factors, the *third power*, etc.

13. Required the second power of 3; of 4; of 6; of 7; of 8: the third power of 2; of 3; of 4; of 6.

14. *One* of the *two equal* factors of a number is the *second root* of a number; *one* of the *three equal* factors is the *third root*, etc.

15. What is the second root of 16? of 25? of 36? of 49? What is the third root of 8? of 27? of 64? of 125?

16. What would it seem natural to call the process of making composite numbers? *Ans. Composition.*

17. What would it seem natural to call the process of finding the factors of a number? *Ans. Factoring.*

18. What are the first four operations of arithmetic called? *Ans.* The *Fundamental* or *Primary Operations* of arithmetic.

19. What would it be natural to call these operations which are derived from the fundamental operations? *Ans* The *Derivative* or *Secondary Operations.*

SECTION III.

SECONDARY OPERATIONS.

100. The **Primary Operations** of arithmetic are those of synthesis and analysis, including the four fundamental rules.

101. The **Secondary, or Derivative Operations,** are those which arise from or grow out of the primary operations of synthesis and analysis.

102. The **Secondary Operations** are *Composition, Factoring, Greatest Common Divisor, Least Common Multiple, Involution,* and *Evolution.*

COMPOSITION.

103. Composition is the process of forming composite numbers when their factors are given.

104. A **Composite Number** is a number which can be produced by multiplying together two or more numbers, each greater than a unit; as 8, 12, 15, etc.

105. The **Factors** of a composite number are the numbers, which, when multiplied together, will produce it; thus 4 and 2 are the factors of 8.

106. A **Prime Number** is one that cannot be produced by multiplying together two or more numbers, each greater than a unit; as, 2, 5, 7, 11, etc.

107. A composite number consisting of two equal factors is said to be the 2d *power* of that factor; of three equal factors, the 3d *power*, etc.; thus, 9 is the 2d power of 3, and 64 is the 3d power of 4.

NOTE.—An even number is one that is exactly divisible by 2; an odd number is one that is not exactly divisible by 2.

MENTAL AND WRITTEN EXERCISES.

1. Tell which of the following numbers are prime or composite: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16.

2. Name the prime numbers from 1 to 53. Name the prime numbers from 53 to 101.

3. Write the numbers from 1 to 100, and cut out all the composite numbers, leaving the primes.

4. What is the second power of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20?

5. What is the 3d power of each of the above? The 4th power? The 5th power? The 6th power?

PRINCIPLES.

1. *Every composite number is equal to the product of its factors.*

2. *A factor of a number is a factor of any number of times that number.*

108. ***To form composite numbers out of any factors.***

1. Form a composite number out of 4, 9, and 5.

SOLUTION.—A composite number formed out of the factors, 4, 9, and 5, is equal to their product, which is 180.

OPERATION.
$4 \times 9 \times 5 = 180$

WRITTEN EXERCISES.

Form composite numbers out

2. Of 5, 6, 7, and 8. *Ans.* 1680.

3. Of two 2's, 3, and 7. *Ans.* 84.

4. Of three 3's, four 2's, and two 5's. *Ans.* 10800.

5. Find a number consisting of four 5's. *Ans.* 625

6. Find the fifth power of 3, of 4, of 7.
Ans. 243; 1024; 16807.

7. Form a composite number out of the first four prime numbers after unity. *Ans.* 210.

8. Form a composite number out of all the prime numbers between 11 and 29. *Ans.* 96577.

9. Form all the composite numbers you can out of 2, 3, 5, and 7. *Ans.* 6; 10; 14; 15; 21; etc.

10. Form all the composite numbers you can out of 2, 3, 5, 7, and 11. *Ans.* 6; 10; 14; 22; 15; 21; etc.

11. Find a composite number consisting of three factors, the first being 2, the second being twice as great, and the third three times as great. *Ans* 48.

DIVISIBILITY OF COMPOSITE NUMBERS.

109. Composite Numbers can be divided by the factors which produce them.

110. The **Factors** of many composite numbers may be seen by inspection from the following principles:

PRINCIPLES.

1. *A number is divisible by* 2 *when the right hand term is zero or an even digit.*

For, the number is evidently an even number, and all even numbers are divisible by 2.

2. *A number is divisible by* 3 *when the sum of the digits is divisible by* 3.

This may be shown by trying several numbers, and, seeing that it is true with these, we infer that it is true with all. A rigid demonstration is too difficult for this place.

3. *A number is divisible by* 4 *when the two right hand terms are ciphers, or when the number they express is divisible by* 4.

If the two right hand terms are ciphers, the number equals a number of hundreds, and since 100 is divisible by 4, any number of hundreds is divisible by 4.

If the number expressed by the two right hand digits is divisible by 4, the number will consist of a number of *hundreds* plus the number expressed by the two right hand digits (thus $1232=1200+32$); and since both of these are divisible by 4, their sum, which is the number itself, is divisible by 4.

4. *A number is divisible by* 5 *when its right hand term is* 0 *or* 5.

When the unit figure is 0, the last partial dividend must be 0, 10, 20, 30, or 40, each of which is divisible by 5. When the unit figure is 5, the last partial dividend must be 15, 25, 35, or 45, each of which is divisible by 5. Therefore, etc.

5. *A number is divisible by* 6 *when it is even, and the sum of the digits is divisible by* 3.

Since the number is even it is divisible by 2, and since the sum of the digits is divisible by 3 the number is divisible by 3, and since it contains both 2 and 3, it will contain their product, 3×2, or 6.

6. *A number is divisible by* 8 *when the three right hand terms are ciphers, or when the number expressed by them is divisible by* 8.

If the three right hand terms are ciphers, the number equals a number of *thousands*, and since 1000 is divisible by 8, any number of thousands is divisible by 8.

If the number expressed by the three right hand digits is divisible by 8, the entire number will consist of a number of *thousands* plus the number expressed by the three right hand digits (thus 17368=17000+368) and since both of these parts are divisible by 8, their sum, which is the number itself, is divisible by 8.

7. *A number is divisible by* 9 *when the sum of the digits is divisible by* 9.

This may be shown by trying several numbers, and, seeing that it is true with these, we can infer that it is true with all. It may also be rigidly demonstrated.

8. *A number is divisible by* 10 *when the unit figure is* 0.

For, such a number equals a number of *tens*, and any number of tens is divisible by 10, hence the number is divisible by 10.

NOTE.—1. *A number is divisible by* 7 *when the sum of the odd numerical periods, minus the sum of the even numerical periods, is divisible by* 7.

2. *A number is divisible by* 11 *when the difference between the sums of the digits in the odd places and in the even places is divisible by* 11, *or when this difference is* 0.

3 These two principles are rather curious than useful. For their demonstration see *Higher Arithmetic.*

INTRODUCTION TO FACTORING.

MENTAL EXERCISES.

1. Name the prime numbers from 1 to 50.

2. Name the composite numbers from 0 to 50.

3. Name some of the factors or makers of 12, 15, 21, 28, 36, 54, 72.

4. Name the prime numbers which are factors of 12, 18, 20, 24, 36, 54, and 72.

5. What shall we call the factors of numbers when they are prime numbers? *Ans.* The *Prime Factors.*

6. Name the prime factors of 12, 16, 18, 20, 24, 30, 32, 36, 40, 45, 50, 60, and 80.

7. Illustrate the principle that the factors of a number are *divisors* of the number.

8. How then can we find the factors of a number? *Ans.* By finding the divisors of a number.

9. How can we find the divisors of a number? *Ans.* By trial, aided by the principles of Art. 110.

10. What do we call that subject of arithmetic which treats of finding the factors of numbers? *Ans. Factoring.*

11. How then may we define the subject of Factoring?

FACTORING.

111. Factoring is the process of finding the factors of composite numbers. Unity and the number itself are not regarded as factors.

112. The **Factors** of a composite number are the numbers which multiplied together will produce it; thus, 3 and 4 are factors of 12.

113. The **Prime Factors** of a composite number are the prime numbers which multiplied together will produce it; thus, 2, 2, and 3 are the prime factors of 12.

114. One of the two equal factors of a number is called its 2d, or *square root;* one of the 3 equal factors, its 3d, or *cube root,* etc.; thus, 3 is the 2d root of 9, 2 the 3d root of 8.

PRINCIPLES.

1. *A divisor of a number, excepting unity and itself, is a factor of that number.*

2. *A divisor of a factor of a number, excepting unity, is a factor of the number.*

3. *A number is divisible by its prime factors or by any product of them.*

4. *A number is divisible only by its prime factors or by some product of them, or by unity.*

CASE I.

115. ***To resolve a number into its prime factors.***

1. Find the prime factors of 105.

SOLUTION.—Dividing by 3, we find that 3 is a factor of 105 (Prin. 1). Dividing the quotient by 5, we find that 5 and 7 are factors of 35 (Prin. 2), and since these numbers 3, 5, and 7, are prime numbers, they are the prime factors of 105.

OPERATION.

3)105
5)35
7

Rule.—I. *Divide the given number by any prime number greater than* 1, *that will exactly divide it.*

II. *Divide the quotient, if composite, in the same manner, and thus continue until the quotient is prime.*

III. *The divisors and last quotient will be the prime factors required.*

What are the prime factors

2. Of 28? *Ans.* 2, 2, 7.
3. Of 84? *Ans.* 2, 2, 3, 7.
4. Of 125? *Ans.* 5, 5, 5
5. Of 325? *Ans.* 5, 5, 13.
6. Of 210? *Ans.* 2, 3, 5, 7.
7. Of 114? *Ans.* 2, 3, 19.
8. Of 432? *Ans.* 2, 2, 2, etc.
9. Of 426? *Ans.* 2, 3, 71.
10. Of 1872? *Ans.* 2, 2, 2, 2, 3, 3, 13.
11. Of 7644? *Ans.* 2, 2, 3, 7, 7, 13.
12. Of 1184? *Ans.* 2, 2, 2, 2, 2, 37.
13. Of 1140? *Ans.* 2, 2, 3, 5, 19.

CASE II.

116. ***To resolve a number into equal factors.***

1. Find the two equal factors of 1225.

SOLUTION.—We first resolve the number into its prime factors. Now since there are *two* 5's, we take *one* 5 for each factor; and since there are *two* 7's, we take *one* 7 for each factor; hence each of the two equal factors is 5×7, or 35. Therefore 35 is one of the two equal factors of 1225.

OPERATION.

$$\begin{array}{r} 5)1225 \\ 5)245 \\ 7)49 \\ \hline 7 \end{array}$$

$5\times7=35$

Rule.—I. *Resolve the number into its prime factors.*

II. *Take the continued product, of one of each of the two equal factors, when we wish the two equal factors, one of each of the three, for the three equal factors, etc.*

2. Find the two equal factors of 16, 36, 100, 196, 256, 324, 900, 1296, 2025. *Ans.* 4, 4; 6, 6; 10, 10; 14, 14, etc.

3. Find the square root of 225, 576, 1764, 3136, 3969, 5184

Ans. 15; 24; 42; 56; 63; 72.

4. Find the cube root of 27, 64, 125, 216, 512, 1000.

Ans. 3; 4; 5; 6; 8; 10.

5. Find the fourth root of 16, 81, 256, 1296, 4096, 20736

Ans. 2; 3; 4; 6; 8; 12.

6. Find the fifth root of 32; 243; 1024; 3125.

Ans. 2; 3; 4; 5.

INTRODUCTION TO COMMON DIVISOR.

MENTAL EXERCISES.

1. Name an exact divisor of 6; of 9; of 12; of 15; of 18; of 20; of 24; of 36.

2. What exact divisors are common to 8 and 12? to 10 and 15? to 12 and 18? to 24 and 36? to 32 and 48? to 48 and 72?

3. What may a divisor *common* to two or more numbers be called? *Ans.* Their *common divisor.*

4. What is a common divisor of 16 and 24? of 15 and 20? of 18 and 30? of 48 and 54?

5. What is the greatest divisor common to 24 and 32? to 32 and 56? to 48 and 72? to 72 and 96?

6. What may the greatest divisor common to two or more numbers be called? *Ans.* Their *greatest common divisor.*

7. What is the greatest common divisor of 24 and 30? of 45 and 50? of 54 and 60? of 64 and 72?

8. What prime factors are common to 18 and 24? 27 and 30? 30 and 35? 36 and 40?

9. The product of what two prime factors of 12 and 18 will divide both? of 20 and 30?

GREATEST COMMON DIVISOR.

117. A Divisor of a number is a number that exactly divides it. Thus 4 is a divisor of 20.

118. A Common Divisor of two or more numbers is a number that exactly divides each of them. Thus 4 is a common divisor of 16 and 20.

119. The **Greatest Common Divisor** of two or more numbers is the greatest number that exactly divides each of them. Thus 8 is the greatest common divisor of 16 and 24.

NOTE.—The greatest common divisor may be represented by the initials G. C. D.

PRINCIPLES.

1. *A common factor of two or more numbers is a factor of their greatest common divisor.*

2. *The product of all the common prime factors of two or more numbers is their greatest common divisor.*

3. *A common divisor of two numbers is a divisor of their sum and also of their difference.*

DEM.—Take any two numbers, as 12 and 20, of which 4 is a common divisor. Now, 12 equals *three* times 4, and 20 equals *five* times 4, and their sum equals *three* times 4 plus *five* times 4, or *eight* times 4, which contains 4, or is divisible by 4. Their difference is *five* times 4 minus *three* times 4, or *two* times 4, which is also divisible by 4.

CASE I.

120. ***When the numbers are small and can be readily factored.***

FIRST METHOD.

121. This method consists in finding the common factors, and taking their product.

1. Find the greatest common divisor of 42, 84, and 126.

SOLUTION.—We write the numbers one beside another, as in the margin. Dividing by 2, we see that 2 is a factor of each number; it is therefore a factor of the G. C. D. (Prin. 1). Dividing the quotients by 3, we see that 3 is a factor of each number and therefore a factor of the G. C. D.; and in the same way we see that 7 is a factor of the G. C. D. Now since the quotients 1, 2, and 3 are prime to each other, 2, 3, and 7 are all the common factors; hence their product, which is 42, is the G. C. D. (Prin. 2).

OPERATION.

2	42 - 84 - 126
3	21 - 42 - 63
7	7 - 14 - 21
	1 - 2 - 3

$2 \times 3 \times 7 = 42$

Rule.—I. *Write the numbers one beside another, with a vertical line at the left, and divide by any common factor of all the numbers.*

II. *Divide the quotients in the same manner, and thus continue until the quotients have no common factor.*

III. *Take the product of all the divisors; the result will be the greatest common divisor.*

What is the greatest common divisor of

2. 10, 20, and 30? *Ans.* 10.
3. 36, 48, and 54? *Ans.* 6.
4. 18, 36, and 72? *Ans.* 18.
5. 48, 72, and 96? *Ans.* 24.
6. 120, 210, and 360? *Ans.* 30.
7. 210, 315, and 420? *Ans.* 105.
8. 252, 336, and 420? *Ans.* 84.
9. 330, 495, and 660? *Ans.* 165.
10. 468, 780, and 1092? *Ans.* 156.

SECOND METHOD.

122. This method consists in resolving the numbers into their prime factors, and taking the product of the common factors.

1. Find the greatest common divisor of 42, 84, and 126.

SOLUTION.—The factors of 42 are 2, 3, and 7; the factors of 84 are 2, 2, 3, and 7; the factors of 126 are 2, 3, 3, and 7. We see that 2, 3, and 7 are all the prime factors common to the three numbers; hence their product, which is 42, is the greatest common divisor of the numbers (Prin. 2). Hence the following

OPERATION.

$42=2\times3\times7$
$84=2\times2\times3\times7$
$126=2\times3\times3\times7$
$2\times3\times7=42$

Rule.—*Resolve the numbers into their prime factors, and take the product of all the common factors.*

Find the greatest common divisor of

2. 270, 315, and 405. *Ans.* 45.
3. 168, 192, and 216. *Ans.* 24.
4. 252, 308, and 364. *Ans* 28.
5. 504, 546, and 588. *Ans* 42.
6. 392, 448, and 504. *Ans.* 56.
7. 432, 504, and 648. *Ans.* 72.
8. 792, 864, 936, and 1008. *Ans.* 72.
9. 384, 576, 768, and 960. *Ans.* 192.

CASE II.

123. ***When the numbers are large and cannot be readily factored.***

1. Find the greatest common divisor of 32 and 56:

SOLUTION.—We divide 56 by 32, the divisor 32 by 24, and the divisor 24 by the remainder 8, and have no remainder; then will 8 be the greatest common divisor of 32 and 56. For

OPERATION.

32)56(1
32
24)32(1
24
8)24(3
24

1st. *The last remainder*, 8, *is a* NUMBER OF TIMES *the G. C. D.* Since 32 and 56 are each a number of times the G. C. D., their difference, 24, is a number of times the G. C. D., by Prin. 3; and since 24 and 32 are each a number of times the G. C. D., their difference, 8, is also a number of times the G. C. D.

2d. *The last divisor*, 8, *is* ONCE *the G. C. D.* Since 8 divides 24 it will divide 24+8, or 32, by Prin. 3; and since it divides 32 and 24, it will divide 32+24, or 56; and now since 8 divides 32 and 56, and is a *number of times the G. C. D.*, it must be *once* the G. C. D.

ANOTHER FORM.—In the margin on the right is another form of writing the division, which in practice we prefer to the above. The problem is to find the greatest common divisor of 32 and 116. The method will be clear from a slight inspection of the work. Let the pupils adopt it after they are familiar with the common form.

OPERATION.

32	116	3
	96	
20	20	1
12	12	1
8	8	1
4	8	2

Rule.—*Divide the greater number by the less, the divisor by the remainder, and thus continue to divide the last divisor by the last remainder until there is no remainder; the last divisor will be the greatest common divisor.*

NOTE.—To find the greatest common divisor of more than two numbers, we first find the greatest common divisor of two of them, then of that divisor and one of the other numbers, etc.

WRITTEN EXERCISES.

Find the greatest common divisor of

2. 115 and 161. *Ans.* 23.
3. 91 and 143. *Ans.* 13.
4. 333 and 592. *Ans.* 37.
5. 697 and 820. *Ans.* 41.

6. A farmer has two heaps of apples, one containing 364, and the other 585, which he wishes to divide into smaller heaps, each containing the same number; what is the largest number that the heaps may contain? *Ans.* 13.

7. A benevolent society distributed $678, $906, and $1146 in equal sums to the poor of three wards of a city, the sums being as large as possible. Required the amount of the equal sums and the number of persons receiving relief in each ward?

Ans. $6; 113; 151; 191.

SUPPLEMENTARY PROBLEMS.

To be omitted unless otherwise directed.

8. 1220 and 2013. *Ans.* 61.
9. 730, 1241, and 1460. *Ans.* 73.
10. 72491 and 103121. *Ans.* 1021.
11. 347387 and 561851. *Ans.* 1117.

12. A Western landholder has three tracts, the first containing 533 acres, the second 574 acres, and the third 861 acres, which he wishes to divide into fields of equal size, having the least number possible. Required the number of fields and the number of acres in each. *Ans.* 48 fields; 41 acres.

INTRODUCTION TO COMMON MULTIPLE.

MENTAL EXERCISES.

1. What number is three times 5? four times 6? five times 6? six times 8?

2. A number which is one or more times another number is called a *multiple* of that number.

3. What is the multiple of 4? of 5? of 6? of 7? of 8? of 9? of 10? of 11? of 12?

4. What multiple is common to 2 and 3? to 3 and 4? to 4 and 6? to 6 and 8? to 6 and 9?

5. What may we call a multiple common to two or more numbers? *Ans.* A *common multiple.*

6. What is a common multiple of 4 and 5? 8 and 9? 6 and 7? 4 and 6? 5 and 8?

7. What is the least multiple common to 2 and 4? 4 and 6? 4 and 8? 6 and 8? 8 and 12?

8. What shall we call the least multiple common to two or more numbers? *Ans.* Their *least common multiple.*

9. What is the least common multiple of 4 and 6? 9 and 12? 10 and 16? 20 and 24? 25 and 30?

LEAST COMMON MULTIPLE.

124. A **Multiple** of a number is one or more times the number; thus, 4 times 5, or 20, is a multiple of 5.

125. A **Common Multiple** of two or more numbers is a number which is a multiple of each of them; thus, 24 is a common multiple of 2, 3, and 4.

126. The **Least Common Multiple** of two or more numbers is the least number which is a multiple of each of them; thus, 12 is the least common multiple of 2, 3, and 4.

NOTE.—The least common multiple may be represented by the initials L. C. M.

PRINCIPLES.

1. *A multiple of a number is exactly divisible by that number.*

2. *A multiple of a number must contain all the prime factors of that number.*

3. *A common multiple of two or more numbers must contain all the prime factors of each of those numbers.*

4. *The least common multiple of two or more numbers must contain all the prime factors of each number, and no other factors.*

CASE I.

127. ***When the numbers are small and easily factored.***

FIRST METHOD.

128. This method consists in resolving the numbers into their prime factors, and taking the product of all the different factors.

1. Find the least common multiple of 12, 30, and 70.

SOLUTION.—We first resolve the numbers into their prime factors. A multiple of 12 must contain the factors of 12, 2, 2, 3; a multiple of 30 must contain the factors of 30, 2, 3, 5; a multiple of 70 must contain the factors of 70, 2, 5, 7; hence the common multiple of 12, 30, and 70 must contain all these different factors and no others; therefore $2\times2\times5\times3\times7$, or 420, is the L. C. M. of 12, 30, and 70 (Prin. 4).

OPERATION.

$12=2\times2\times3$
$30=2\times3\times5$
$70=2\times5\times7$
$2\times2\times3\times5\times7=420$

Rule.—I. *Resolve the numbers into their prime factors.*

II. *Take the product of all the different factors, using each factor the greatest number of times it occurs in either number.*

NOTE.—Any numbers which are divisors of the others may be omitted, since the multiple of the other numbers will be a multiple of these.

Find the least common multiple of

2. 24, 30, and 36. *Ans.* 360.
3. 16, 24, and 56. *Ans.* 336.
4. 28, 36, and 60. *Ans.* 1260.
5. 36, 48, and 84. *Ans.* 1008.
6. 63, 72, and 108. *Ans.* 1512.
7. 15, 30, 42, and 72. *Ans.* 2520.
8. 22, 55, 77, and 110. *Ans.* 770.
9. 33, 99, 36, 108, and 135. *Ans.* 5940.
10. A has \$14, B \$15, C \$36, and D as many as the least common multiple of the amounts of the others; how many has D? *Ans.* \$1260.

SECOND METHOD.

129. This method consists in taking out the prime factors of the least common multiple and finding their product.

1. Find the least common multiple of 12, 30, and 70.

SOLUTION.—Placing the numbers one beside another and dividing by 2, we see that 2 is a factor of each of them, it is therefore a factor of the L. C. M. (Prin. 3); dividing the quotients that will contain it by 3, we see that 3 is a factor of some of the numbers, it is therefore a factor of the L. C. M. Dividing the next quotients by 5, we see that 5 is a factor of some of them, hence 5 is a factor of the L. C. M.; and the quotients having no other common factor, we see that the factors of the given numbers are 2, 3, 5, 2, and 7, hence their product, which is 420, is the L. C. M. Hence the following

OPERATION.

2	12 - 30 - 70
3	6 - 15 - 35
5	2 - 5 - 35
	2 - 1 - 7

$2\times3\times5\times2\times7=420$

Rule.—I. *Write the numbers one beside another, divide by any prime number that will exactly divide two or more, and write the quotients and undivided numbers beneath.*

II. *Divide the quotients in the same manner, and thus continue until no two numbers in the lowest line have a common factor.*

III. *Take the product of the divisors and final quotients; the result will be the least common multiple required.*

Find the least common multiple of

2. 16, 20, and 30. *Ans.* 240.
3. 28, 56, and 84. *Ans.* 168.
4. 48, 60, and 30. *Ans.* 240.
5. 150, 200, and 250. *Ans.* 3000.
6. 40, 96, 100, and 120. *Ans.* 2400.
7. 120, 180, 200, and 240. *Ans.* 3600.
8. 140, 280, 160, and 320. *Ans.* 2240.

CASE II.

130. ***When the numbers are large and cannot be readily factored.***

1. Find the least common multiple of 28 and 63.

SOLUTION.—The greatest common divisor of these numbers is 7; 28 equals 4 times 7, and 63 equals 9 times 7; hence the L. C. M., as found in the first method, is $4\times7\times9$, which equals 28 multiplied by 63 divided by 7; or *the first number multiplied by the second divided by their greatest common divisor.*

OPERATION.

$28=4\times7$; $63=9\times7$

L. C. M. $=4\times7\times9$

$=28\times\frac{63}{7}$

Rule.—I. *Find the greatest common divisor of two numbers, divide one number by it, and multiply the other number by the quotient.*

II. *When there are more than two numbers, find the least common multiple of two of the numbers, and then of this number and the third number,* etc.

WRITTEN EXERCISES.

Find the least common multiple of

2. 671 and 793. *Ans.* 8723.
3. 3503 and 4859. *Ans.* 150629.
4. 6527 and 7597. *Ans.* 463417.

5. What is the smallest sum of money for which I could hire workmen for one month, paying either \$16, \$20, \$28, or \$35 a month? *Ans.* \$560.

6. A can dig 14 rods of ditch in a week, B can dig 18 rods, C 22 rods, and D 24 rods; what is the least number of rods that would afford an exact number of weeks' work for each one of them? *Ans.* 5544 rods.

7. What is the smallest number of bushels of corn that will fill a number of barrels containing 3 bushels each, a number of sacks containing 5 bushels each, a number of casks containing 14 bushels each, or a number of bins containing 48 bushels each? *Ans.* 1680 bu.

SUPPLEMENTARY PROBLEMS,

To be omitted unless otherwise directed.

8. 7205 and 9432. *Ans.* 518760.
9. 11183 and 15403. *Ans.* 816359.
10. 357, 612, and 663. *Ans.* 55692.
11. 4141, 6161, and 7171. *Ans.* 17934671.

12. Four men start at the same place to walk around a garden; A can go around in nine minutes, B in 10 minutes, C in 12 minutes, and D in 15 minutes; in what time will they all meet at the starting point? *Ans.* 180 minutes.

13. A, B, C, and D start from the same point, A traveling a mile in 18 minutes, B in 24 minutes, C in 30 minutes, and D in 35 minutes; what is the least whole number of miles each may travel that they may return to the starting point at the same moment?

Ans. A, 140; B, 105; C, 84; D, 72.

INTRODUCTION TO CANCELLATION.

MENTAL EXERCISES.

1. If we omit the factor 2 from 12 and 6, what factors will remain?

2. Divide 24 by 6. Divide 24 by $\frac{1}{2}$ of 6. Divide $\frac{1}{2}$ of 24 by 6.

3. Divide $\frac{1}{2}$ of 24 by $\frac{1}{2}$ of 6. Divide 36 by 18, first taking out the common factor 6.

4. Is there any difference in the quotient of 48 divided by 12, and $\frac{1}{2}$ of 48 divided by $\frac{1}{2}$ of 12?

5. Divide 72 by 48, first omitting common factors. Divide 90 by 60 in the same way; 144 by 96.

6. Divide $2\times2\times2$ by 2×2; $3\times3\times4$ by 2×3; $3\times4\times5$ by 3×5.

7. Divide $2\times3\times7$ by 2×7; $2\times3\times4$ by 2×3; $3\times5\times8$ by 3×8; $6\times5\times3$ by 3×6; $2\times7\times9\times10$ by 9×2.

CANCELLATION.

131. Cancellation is a process of abbreviating arithmetical operations by rejecting common factors in both dividend and divisor.

PRINCIPLES.

1. *The cancelling of a factor from any number divides the number by that factor.*

DEM.—Thus if we take the factor 3 out of 24 we shall divide 24 by 3.

2. *The cancelling of a factor in both dividend and divisor will not change the quotient.*

DEM.—Cancelling a factor in both dividend and divisor is the same as dividing them both by the same number, which, by the principles of division, does not change the quotient.

1. Divide 84×60 by 24×63.

SOLUTION.—We cancel the common factor 12 from 60 and 24, writing 5, the other factor of 60, above 60, and 2, the other factor of 24, below 24; we then cancel the common factor 21 from 84 and 63, writing 4, the other factor of 84, above 84, and 3, the other factor of 63, below 63; we then cancel 2 from 2 and 4, writing 2 above the 4; the product of the remaining factors of the dividend is 10, the product of the remaining factors of the divisor is 3; hence the quotient is 10 divided by 3, or $3\frac{1}{3}$.

OPERATION.

$$\frac{\overset{4}{\cancel{84}}\times\overset{5}{\cancel{60}}}{\underset{2}{\cancel{24}}\times\underset{3}{\cancel{63}}}=\frac{10}{3}=3\tfrac{1}{3}$$

Rule.—I. *Cancel the common factors from the dividend and divisor.*

II. *Then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.*

NOTES.—1. The unit 1 takes the place of a cancelled factor, but need not be written, except in the dividend of the quotient, when there are no other factors of the dividend remaining.

2. A factor in one term will cancel two or more factors in the other term, when their product is equal to the former.

3. Some prefer to place the dividend upon the right and the divisor upon the left, of a vertical line.

2. Divide $12\times14\times16$ by $6\times7\times8$. *Ans.* 8.

3. Divide $20\times32\times35$ by $4\times5\times16$. *Ans.* 70.

4. Divide 125×250 by $15\times50\times75$. *Ans.* $\frac{5}{9}$.

5. Divide 180×270 by 45×108. *Ans.* 10.

6. Divide 120×140 by 60×350. *Ans.* $\frac{4}{5}$.

7. Divide $45\times49\times81$ by $35\times84\times63$. *Ans.* $\frac{27}{28}$.

8. Divide $60\times77\times320$ by $25\times42\times33$. *Ans.* $42\frac{8}{9}$.

9. Divide $75\times42\times99$ by $125\times63\times33$. *Ans.* $1\frac{1}{5}$.

WRITTEN EXERCISES.

1. How many yards of muslin, worth 12 cents a yard, may be bought for 16 pounds of butter, worth 15 cents a pound?

SOLUTION.—If one pound of butter is worth 15 cents, 16 pounds are worth 16×15 cents; for 16×15 cents, at 12 cents a yard, we can get as many yards of muslin as 12 is contained times in 15×16, which we find, by cancellation, to be 20.

OPERATION.

$$\frac{\overset{5}{\cancel{15}}\times\overset{4}{\cancel{16}}}{\underset{4}{\cancel{12}}}=20$$

2. How many bushels of corn, worth 45 cents a bushel, must be exchanged for 125 pounds of butter, at 18 cents a pound? *Ans.* 50.

3. A exchanged rye, worth 84 cents per bushel, for 78 bushels of wheat, worth 98 cents per bushel; required the number of bushels of rye. *Ans.* 91.

4. How many bushels of corn, at 42 cents a bushel, must be given in exchange for 7 pieces of cloth, each containing 40 yards, at 36 cents a yard? *Ans.* 240.

5. How many boxes of tea, each containing 24 pounds at 90 cents a pound, must be given for 27 firkins of butter of 56 pounds each, at 20 cents a pound? *Ans.* 14.

6. A farmer sold a grocer 9 loads of apples, each load containing 18 bags; and each bag 2 bushels, at 35 cents a bushel, and received in payment 12 boxes of sugar, each containing 135 pounds; what was the sugar worth a pound?

Ans. 7 cents.

PRIME NUMBERS.

132. No general method of determining prime numbers, beyond a certain limit, has yet been discovered, although much time has been spent in the investigation.

133. We give the following practical method, which consists in writing a series of numbers, and sifting out those which are composite.

METHOD.—Since the even numbers after 2 are composite, we write the series of odd numbers; thus,

1, 3, 5, 7, ^{3}9, 11, 13, ^{3,5}15, 17, 19, ^{3,7}21, 23, ^{5}25, ^{3}27, 29, 31, ^{3}33, ^{5,7}35, 37, ^{3}39, 41,
^{3,5}43, 45, ^{7}47, ^{3}49, 51, ^{5}53, ^{3}55, 57, ^{3,7}59, 61, ^{5}63, 65, ^{3}67, 69, ^{3,5}71, 73, ^{7}75, ^{3}77, 79, 81,
^{5}83, ^{3}85, ^{7}87, 89, ^{3}91, 93, ^{5}95, ^{3}97, 99.

Now, commencing at 3, since every third term is divisible by 3, every third number is composite, which we indicate by putting the figure 3 over it.

Commencing at 5, every fifth number is divisible by 5, and is therefore composite, hence we place a figure 5 over every fifth number. Proceed in the same manner with 7, and the numbers unmarked will be the prime numbers up to 100.

This method was discovered by Eratosthenes, a Greek mathematician. He inscribed the series of odd numbers on parchment, and then cutting out the composite numbers, his parchment with its holes resembled a sieve; hence the method has been called *Eratosthenes' Sieve.*

TABLE OF PRIME NUMBERS.

1	47	113	197	281	379	463	571	659	761	863
2	53	127	199	283	383	467	577	661	769	877
3	59	131	211	293	389	479	587	673	773	881
5	61	137	223	307	397	487	593	677	787	883
7	67	139	227	311	401	491	599	683	797	887
11	71	149	229	313	409	499	601	691	809	907
13	73	151	233	317	419	503	607	701	811	911
17	79	157	239	331	421	509	613	709	821	919
19	83	163	241	337	431	521	617	719	823	927
23	89	167	251	347	433	523	619	727	827	937
29	97	173	257	349	439	541	631	733	829	941
31	101	179	263	353	443	547	641	739	839	947
37	103	181	269	359	449	557	643	743	853	953
41	107	191	271	367	457	563	647	751	857	967
43	109	193	277	373	461	569	653	757	859	971

NOTE.—This page will be of interest to the student to read, but is not to be recited.

INTRODUCTION TO FRACTIONS.

MENTAL EXERCISES.

1. If an apple is divided into two equal parts, what is one of these parts called?

2. What are two of these parts called? How many halves in anything?

3. What is $\frac{1}{2}$ of 6? of 4? of 12? of 16? of 10? of 18? of 20? of 24? of 28? of 36?

4. If I divide an apple into 3 equal parts, what is one of these parts called?

5. What are 2 and 3 of these parts called? How many thirds in anything?

6. The number of equal parts into which a unit may be divided is represented by a figure below the line; thus $\frac{1}{2}$ represents *halves*; $\frac{1}{3}$, *thirds*; $\frac{1}{4}$, *fourths*, etc.

7. The number of fractional parts taken may be represented by a figure above the line; thus, $\frac{2}{3}$ represents 2 *thirds*; $\frac{3}{4}$, 3 *fourths*; $\frac{5}{6}$, 5 *sixths*, etc.

8. What is $\frac{1}{2}$ of 6? of 9? of 12? of 18? of 15? of 21? of 27? What are $\frac{2}{3}$ of 12? of 15? of 21? of 18? of 24? of 33?

9. If I divide an apple into 4 equal parts, what is one of these parts called? If I divide in 5, or 6, etc., equal parts?

10. How many fourths make a whole? How many fifths? Sixths? Sevenths? Eighths? Ninths? Tenths?

11. What is $\frac{1}{4}$ of 12? $\frac{1}{5}$ of 20? $\frac{1}{8}$ of 24? $\frac{3}{4}$ of 16? $\frac{2}{5}$ of 30? $\frac{1}{7}$ of 28? $\frac{3}{8}$ of 40? $\frac{1}{7}$ of 35?

12. What is $\frac{3}{4}$ of 20? $\frac{2}{3}$ of 15? $\frac{5}{8}$ of 12? $\frac{7}{8}$ of 24? $\frac{4}{5}$ of 27? $\frac{5}{6}$ of 30? $\frac{4}{11}$ of 22? $\frac{7}{8}$ of 64?

13. If a yard of muslin cost 24 cents, what will $\frac{1}{4}$ of a yard cost? What will $\frac{3}{4}$ of a yard cost?

14. Henry's age is 36 years, and his wife's age is $\frac{5}{6}$ as much; what is his wife's age?

15. If 5 melons cost 60 cents, what will 7 melons cost at the same rate?

16. What will 4 yards of satin cost at the rate of \$6 for $\frac{3}{5}$ of a yard?

17. What must I pay for three-fourths of a ton of hay if five-sixths of a ton cost \$20?

18. What will $\frac{7}{8}$ of a ton of coal cost at the rate of \$4.50 for $\frac{3}{4}$ of a ton?

SECTION IV.

COMMON FRACTIONS.

134. A **Fraction** is a number of the equal parts of a unit.

135. **Fractions** are divided into two classes; *common fractions* and *decimal fractions*.

136. A **Common Fraction** is one in which the unit is divided into *any number* of equal parts.

137. A **Decimal Fraction** is one in which the unit is divided into *tenths*, *hundredths*, etc.

138. A **Common Fraction** is expressed by two numbers, one written above the other, with a short line between them; thus, $\frac{3}{4}$ expresses 3 *fourths*.

139. The **Denominator** of a fraction denotes the number of equal parts into which the unit is divided.

140. The **Numerator** of a fraction denotes the number of equal parts which are taken.

141. The **Numerator** and **Denominator** are called the *Terms* of the fraction. The numerator is written above the line, and the denominator below it.

142. **Common Fractions** consist of three principal classes; namely, *Simple*, *Compound*, and *Complex*.

143. A **Simple Fraction** is a fraction having a single integral numerator and denominator; as $\frac{2}{3}$, $\frac{4}{5}$, etc.

144. A **Proper Fraction** is a simple fraction whose value is less than a unit; as $\frac{2}{3}$, $\frac{3}{4}$.

145. An **Improper Fraction** is a simple fraction whose value is equal to or greater than a unit; as $\frac{5}{5}$, $\frac{7}{6}$, $\frac{12}{8}$, etc.

146. A **Compound Fraction** is a fraction of a fraction; as $\frac{2}{3}$ of $\frac{3}{4}$, $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{7}{8}$, etc.

147. A **Complex Fraction** is one whose numerator, or denominator, or both, are fractional; as $\frac{5}{\frac{4}{5}}$, $\frac{\frac{2}{3}\text{ of }\frac{4}{5}}{\frac{3}{4}\text{ of }\frac{5}{6}}$.

148. A **Mixed Number** consists of an integer and a fraction; as, $4\frac{2}{3}$, $5\frac{3}{4}$, etc.

149. The **Reciprocal** of a number is a unit divided by that number; thus, the reciprocal of 3 is $\frac{1}{3}$.

150. The **Number of Cases** of common fractions is eight. They are as follows:

1. Reduction,
2. Addition,
3. Subtraction,
4. Multiplication,
5. Division,
6. Relation of Fractions,
7. Greatest Common Divisor,
8. Least Common Multiple.

NOTES.—1. Each fractional part is used as a single thing and is therefore a unit; hence, we have *Units* and *fractional units*.

2. The primary conception of a fraction is that it is a number of equal parts of a unit. It may, however, be regarded as a *number* of parts of *one* thing, or as *one* part of a *number* of things. Thus, $\frac{2}{3}$ may be regarded as $\frac{2}{3}$ of *one* or $\frac{1}{3}$ of *three*.

NUMERATION AND NOTATION.

151. Numeration of Fractions is the art of reading a fraction when expressed by figures.

Rule.—*Read the number of fractional units expressed by the numerator, and give them the name indicated by the denominator.*

Name the kind and read the following:—

1. $\frac{2}{3}$.
2. $\frac{4}{5}$.
3. $\frac{7}{4}$.
4. $\frac{9}{10}$.
5. $\frac{8}{8}$.
6. $\frac{14}{17}$.
7. $5\frac{3}{7}$.
8. $\frac{18}{16}$.
9. $\frac{4}{5}$ of $\frac{3}{7}$.

152. Notation of Fractions is the art of expressing fractions by means of figures.

Rule.—*Write the number of fractional units, draw a line beneath, under which write the number which indicates the kind of fractional units.*

Write the following fractions:—

1. Two-thirds.
2. Five-sixths.
3. Six-eighths.
4. Nine-tenths.
5. Seven-elevenths.
6. Eight-tenths.
7. Eleven-fifteenths.
8. Twelve-twentieths.

ANALYSIS OF FRACTIONS.

153. To **Analyze** a fraction is to explain what is expressed by the fractional notation.

1. Analyze the fraction $\frac{4}{5}$.

SOLUTION.—In the fraction $\frac{4}{5}$, the denominator 5 indicates that the unit is divided into 5 equal parts, and the numerator 4 denotes that 4 of these parts are taken.

Analyze the following fractions:

2. $\frac{3}{8}$.	5. $\frac{9}{10}$.	8. $\frac{14}{11}$.
3. $\frac{5}{6}$.	6. $\frac{12}{14}$.	9. $\frac{17}{15}$.
4. $\frac{7}{8}$.	7. $\frac{10}{11}$.	10. $\frac{25}{37}$.

METHOD OF TREATMENT.

154. There are **Two Methods** of treating common fractions, which may be distinguished as the *Inductive* and *Deductive Methods.*

155. By the **Inductive Method** we solve all the different cases by analysis, and derive the *rules* or *methods of operation* from these analyses by *inference* or *induction.*

156. By the **Deductive Method** we first establish a few general principles, and then derive all the *rules* or *methods of operation* from these general principles.

NOTE.—The Inductive Method will be used with the mental exercises; with the written exercises the method which is thought to be the simplest is used.

PRINCIPLES OF FRACTIONS.

1. *Multiplying the numerator of a fraction by any number multiplies the value of the fraction by that number.*

If we multiply the numerator of a fraction by any number, as 5, the resulting fraction will express 5 times as many fractional units, each of the same size as before, hence the value of the fraction is 5 times as great.

2. *Dividing the numerator of a fraction by any number divides the value of the fraction by that number.*

If we divide the numerator of a fraction by any number, as 4, the resulting fraction will express $\frac{1}{4}$ as many fractional units, each of the same size as before, hence the value of the fraction is divided by 4.

3. *Multiplying the denominator of a fraction by any number divides the value of the fraction by that number.*

Since the denominator denotes the number of equal parts into which the unit is divided, if we multiply the denominator of a fraction by any number, as 5, the unit will be divided into 5 times as many equal parts, hence each fractional unit will be $\frac{1}{5}$ as large as before, and the same number of fractional units being taken, the value of the fraction is $\frac{1}{5}$ as great.

4. *Dividing the denominator of a fraction by any number multiplies the value of the fraction by that number.*

Since the denominator denotes the number of equal parts into which the unit is divided, if we divide it by any number, as 4, the unit will be divided into 4 as many equal parts, hence each fractional unit will be 4 times as large as before, and the same number of fractional units being taken, the value of the fraction will be 4 times as great.

5. *Multiplying both numerator and denominator of a fraction by the same number does not change the value of the fraction.*

Since multiplying the numerator multiplies the value of the fraction, and multiplying the denominator divides the value of the fraction, multiplying both numerator and denominator both multiplies and divides the value of the fraction by the same number, and hence does not change its value.

6. *Dividing both numerator and denominator of a fraction by the same number does not change its value.*

Since dividing the numerator divides the value of the fraction, and dividing the denominator multiplies the value, dividing both numerator and denominator both divides and multiplies the value of the fraction, and hence does not change its value.

157. These principles may be embodied in one general law as follows:

General Principle.—*A change in the* NUMERATOR *by multiplication or division produces a* SIMILAR *change in the value of the fraction, but such a change in the* DENOMINATOR *produces an* OPPOSITE *change in the value of the fraction.*

REDUCTION OF FRACTIONS.

158. The **Reduction of Fractions** is the process of changing their form without altering their value.

159. There are **Six Cases** of reduction:

1st Numbers to fractions.	4th. To lower terms.
2d. Fractions to numbers.	5th. Compound to simple.
3d. To higher terms.	6th. Complex to simple.

NOTE.—Reducing to a *Common Denominator* is included in these six cases.

CASE I.

160. ***To reduce whole or mixed numbers to improper fractions.***

MENTAL EXERCISES.

1. How many fifths in $4\frac{3}{5}$?

SOLUTION.—In *one* there are 5 *fifths*, and in 4 there are 4 times 5 fifths, or 20 fifths, which added to 3 fifths, equal 23 fifths; therefore $4\frac{3}{5} = \frac{23}{5}$.

2. How many fourths in $7\frac{3}{4}$? in $5\frac{3}{4}$? in $9\frac{1}{4}$? in $12\frac{3}{4}$?

3. How many fifths in $6\frac{3}{5}$? in $7\frac{3}{5}$? in $8\frac{1}{5}$? in $10\frac{4}{5}$?

4. How many sixths in $4\frac{1}{6}$? in $8\frac{5}{6}$? in $9\frac{1}{6}$? in $11\frac{5}{6}$?

5. How many sevenths in $7\frac{4}{7}$? in $9\frac{3}{7}$? in $8\frac{5}{7}$? in $12\frac{6}{7}$?

6. Describe the operation we perform in reducing a mixed number to a fraction.

WRITTEN EXERCISES.

1. Reduce $27\frac{3}{4}$ to fourths.

SOLUTION.—In *one* there are 4 fourths, and in 27 there are 27 times 4 *fourths*, or $\frac{108}{4}$, which added to the $\frac{3}{4}$, equals $\frac{111}{4}$. Therefore, etc.

OPERATION.

$$27\frac{3}{4} = \frac{111}{4}$$

Rule.—*Multiply the whole number by the denominator of the fraction, add the numerator to the product, and write the denominator under the sum.*

Reduce to improper fractions,

2. $5\frac{3}{8}$.	*Ans.* $\frac{43}{8}$.	7. $51\frac{5}{6}$.	*Ans.* $\frac{311}{6}$.
3. $12\frac{3}{5}$.	*Ans.* $\frac{63}{5}$.	8. $35\frac{11}{12}$.	*Ans.* $\frac{431}{12}$.
4. $18\frac{4}{5}$.	*Ans.* $\frac{94}{5}$.	9. $24\frac{7}{18}$; $51\frac{7}{18}$.	*Ans.* $\frac{319}{18}$; $\frac{670}{18}$.
5. $11\frac{5}{9}$.	*Ans.* $\frac{104}{9}$.	10. $82\frac{14}{15}$; $100\frac{99}{100}$.	*Ans.* $\frac{1244}{15}$; $\frac{10099}{100}$.
6. $27\frac{5}{8}$.	*Ans.* $\frac{221}{8}$.	11. $49\frac{49}{50}$; $235\frac{42}{65}$.	*Ans.* $\frac{2499}{50}$; $\frac{15317}{65}$.

CASE II.

161. ***To reduce improper fractions to whole or mixed numbers.***

MENTAL EXERCISES.

1. How many units in $\frac{23}{4}$?

SOLUTION.—In *one* there are $\frac{4}{4}$, hence in 23 fourths there are as many ones as 4 is contained times in 23, which are $5\frac{3}{4}$. Therefore $\frac{23}{4} = 5\frac{3}{4}$.

2. How many units in $\frac{12}{4}$? in $\frac{25}{5}$? in $\frac{23}{6}$?

3. How many units in $\frac{17}{3}$? in $\frac{21}{4}$? in $\frac{25}{6}$?

4. How many units in $\frac{29}{9}$? in $\frac{41}{11}$? in $\frac{45}{12}$?

5. How many units in $\frac{37}{12}$? in $\frac{43}{13}$? in $\frac{63}{15}$?

6. Describe the process of reducing an improper fraction to a mixed number.

WRITTEN EXERCISES.

1. How many units in $\frac{7\,9}{5}$?

SOLUTION.—Since dividing both terms of a fraction by the same number does not change its value (Prin. 6), by dividing both terms by 5, we have $\frac{15\frac{4}{5}}{1}$, or $15\frac{4}{5}$.

OPERATION.
$\frac{7\,9}{5}=15\frac{4}{5}$.

Rule.—*Divide the numerator by the denominator, and the quotient will be the whole or mixed number.*

Reduce to whole or mixed numbers,

2. $\frac{1\,6}{4}$.	*Ans.* 4.	**7.** $\frac{1\,4\,4}{1\,6}$.	*Ans.* 9.
3. $\frac{1\,3}{3}$.	*Ans.* $4\frac{1}{3}$.	**8.** $\frac{2\,9\,8}{1\,6}$.	*Ans.* $18\frac{8}{1\,6}$.
4. $\frac{2\,5}{6}$.	*Ans.* $4\frac{1}{6}$.	**9.** $\frac{2\,8\,3}{1\,5}$.	*Ans.* $65\frac{7}{1\,5}$.
5. $\frac{7\,9}{9}$.	*Ans.* $8\frac{7}{9}$.	**10.** $\frac{1\,2\,0\,8}{1\,4\,8}$.	*Ans.* $8\frac{2\,4}{1\,4\,8}$.
6. $\frac{8\,7}{1\,1}$.	*Ans.* $7\frac{1\,0}{1\,1}$.	**11.** $\frac{1\,0\,0\,0}{4\,0}$.	*Ans.* 25.

12. $\frac{7\,2\,5}{4\,5}$ ft.; $\frac{5\,2\,9}{2\,5}$ yd. *Ans.* $16\frac{5}{4\,5}$; $13\frac{4}{2\,5}$.

13. $\frac{7\,3\,8\,4}{1\,2\,8}$; $\frac{6\,8\,7\,5}{2\,8\,4}$; $\frac{5\,7\,8\,9\,6}{7\,8\,6\,4}$. *Ans.* $59\frac{2\,7}{1\,2\,8}$; $29\frac{8\,9}{2\,8\,4}$; $7\frac{2\,8\,4\,8}{7\,8\,6\,4}$.

CASE III.

162. *To reduce fractions to higher terms.*

163. **Reducing a Fraction to higher terms** is the process of reducing it to an equivalent fraction, having a greater numerator and denominator.

MENTAL EXERCISES.

1. Change $\frac{3}{4}$ to twelfths.

SOLUTION.—In *one* there are $\frac{1\,2}{1\,2}$, and in $\frac{1}{4}$ there are $\frac{1}{4}$ of $\frac{1\,2}{1\,2}$, which are $\frac{3}{1\,2}$ and in $\frac{3}{4}$ there are 2 times $\frac{3}{1\,2}$, which are $\frac{6}{1\,2}$; therefore $\frac{3}{4}=\frac{9}{1\,2}$.

2. Change $\frac{1}{4}$ and $\frac{1}{5}$ to twentieths; $\frac{2}{3}$ and $\frac{3}{5}$ to fifteenths.

3. Change $\frac{1}{5}$ and $\frac{1}{6}$ to thirtieths; $\frac{3}{4}$ and $\frac{7}{8}$ to sixteenths.

4. Change $\frac{2}{3}$ and $\frac{5}{7}$ to seventieths; $\frac{5}{6}$ and $\frac{7}{8}$ to eighteenths.

5. Change $\frac{7}{8}$ and $\frac{3}{5}$ to seventy-seconds; $\frac{9}{1\,0}$ and $\frac{1\,1}{1\,2}$ to 120ths.

6. Describe the process of reducing a fraction to higher terms.

WRITTEN EXERCISES.

1. How many twentieths in $\frac{3}{5}$?

SOLUTION.—Since multiplying both terms of a fraction by the same number does not change its value (Prin. 5), we multiply both terms by the number which will give the required denominator, which we see is 4; hence, $\frac{3}{5}=\frac{1\,2}{2\,0}$.

OPERATION.
$\frac{3}{5}=\frac{3\times4}{5\times4}=\frac{1\,2}{2\,0}$.

Rule.—*Multiply both numerator and denominator by the number which will give the required denominator.*

2. Reduce $\frac{3}{4}$, $\frac{2}{3}$, and $\frac{5}{6}$ to twelfths. *Ans.* $\frac{9}{12}$, $\frac{8}{12}$, $\frac{10}{12}$.
3. Reduce $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{9}{10}$ to twentieths. *Ans.* $\frac{15}{20}$, $\frac{16}{20}$, $\frac{18}{20}$.
4. Reduce $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$ to sixteenths. *Ans.* $\frac{8}{16}$, $\frac{12}{16}$, $\frac{10}{16}$.
5. Reduce $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{7}{10}$ to thirtieths. *Ans.* $\frac{24}{30}$, $\frac{25}{30}$, $\frac{21}{30}$.
6. Reduce $\frac{5}{8}$, $\frac{4}{6}$, and $1\frac{7}{24}$ to forty-eighths. *Ans.* $\frac{36}{48}$, $\frac{32}{48}$, $\frac{34}{48}$.
7. Reduce $\frac{4}{5}$, $\frac{7}{8}$, $\frac{9}{10}$, and $1\frac{1}{12}$ to 120ths.

Ans. $\frac{96}{120}$, $\frac{105}{120}$, $\frac{108}{120}$, $\frac{110}{120}$.

CASE IV.

164. ***To reduce fractions to lower terms.***

165. Reducing a Fraction to lower terms is the process of reducing it to an equivalent fraction having a smaller numerator and denominator.

Principle.—*A fraction is in its lowest terms when the numerator and denominator are prime to each other.*

MENTAL EXERCISES.

1. Reduce $\frac{9}{12}$ to fourths.

SOLUTION.—*One* equals $\frac{12}{12}$, and $\frac{1}{4}$ equals $\frac{3}{12}$, or $\frac{3}{12}$; since $\frac{3}{12}$ equals $\frac{1}{4}$, $\frac{9}{12}$ equals as many fourths as 3 is contained times in 9, which is 3; hence $\frac{9}{12}$ equals $\frac{3}{4}$. Therefore, etc.

Reduce

2. $\frac{15}{20}$ to 4ths. *Ans.* $\frac{3}{4}$.
3. $\frac{15}{20}$ to 5ths. *Ans.* $\frac{3}{4}$.
4. $\frac{3}{24}$ to 6ths. *Ans.* $\frac{3}{4}$.
5. $\frac{54}{63}$ to 7ths. *Ans.* $\frac{6}{7}$.
6. $\frac{54}{72}$ to 8ths. *Ans.* $\frac{6}{8}$.
7. $\frac{48}{108}$, $\frac{84}{126}$ to 9ths. *Ans.* $\frac{4}{9}$, $\frac{6}{9}$

8. Describe the process of reducing a fraction to lower terms.

WRITTEN EXERCISES.

1. Reduce $\frac{24}{60}$ to fifths.

SOLUTION.—Since dividing both terms of a fraction by the same number does not change its value (Prin. 6), we may reduce $\frac{24}{60}$ to lower terms by dividing both numerator and denominator by 6; dividing, we have $\frac{24}{60}$ equal to $\frac{4}{5}$; and since the terms 4 and 5 are prime to each other, the fraction is in its lowest terms. Therefore, etc.

OPERATION.
$6)\frac{24}{60}=\frac{4}{5}$.

Rule I.—*Divide both terms successively by their common factors.*

Rule II.—*Divide both terms by their greatest common divisor.*

Reduce the following fractions to lowest terms:—

2. $\frac{14}{21}$, $\frac{15}{18}$.	*Ans.* $\frac{2}{3}$, $\frac{5}{6}$.		7. $\frac{120}{168}$, $\frac{648}{720}$.	*Ans.* $\frac{5}{7}$, $\frac{9}{10}$.	
3. $\frac{16}{24}$, $\frac{18}{30}$.	*Ans.* $\frac{2}{3}$, $\frac{3}{5}$.		8. $\frac{792}{864}$, $\frac{192}{216}$.	*Ans.* $\frac{11}{12}$, $\frac{8}{9}$.	
4. $\frac{15}{25}$, $\frac{18}{36}$.	*Ans.* $\frac{3}{5}$, $\frac{1}{2}$.		9. $\frac{840}{4812}$, $\frac{1176}{1512}$.	*Ans.* $\frac{15}{77}$, $\frac{7}{9}$.	
5. $\frac{21}{84}$, $\frac{24}{96}$.	*Ans.* $\frac{1}{4}$, $\frac{2}{3}$.		10. $\frac{726}{792}$, $\frac{95540}{188356}$.	*Ans.* $\frac{11}{12}$, $\frac{5}{7}$.	
6. $\frac{72}{288}$, $\frac{288}{864}$.	*Ans.* $\frac{1}{4}$, $\frac{1}{3}$.		11. $\frac{6161}{7171}$, $\frac{425521}{1183861}$.	*Ans.* $\frac{61}{71}$, $\frac{101}{281}$.	

CASE V.

166. ***To reduce compound fractions to simple ones.***

MENTAL EXERCISES.

1. What is $\frac{1}{3}$ of $\frac{1}{4}$?

SOLUTION.—$\frac{1}{3}$ of $\frac{1}{4}$ is *one* of the *three* equal parts into which $\frac{1}{4}$ may be divided; if each fourth is divided into 3 equal parts, 4 fourths, or the unit, will be divided into 4 times 3, or 12 equal parts; hence each part is $\frac{1}{12}$ of a unit. Therefore, $\frac{1}{3}$ of $\frac{1}{4}$ is $\frac{1}{12}$.

2. What is $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{4}$ of $\frac{1}{2}$? $\frac{1}{5}$ of $\frac{1}{7}$?
3. What is $\frac{1}{3}$ of $\frac{1}{5}$? $\frac{1}{3}$ of $\frac{1}{10}$? $\frac{1}{11}$ of $\frac{1}{12}$?
4. What is $\frac{2}{3}$ of $\frac{1}{5}$? $\frac{2}{4}$ of $\frac{2}{3}$? $\frac{2}{5}$ of $\frac{5}{6}$?
5. What is $\frac{3}{5}$ of $\frac{4}{7}$? $\frac{5}{6}$ of $\frac{5}{7}$? $\frac{4}{5}$ of $\frac{6}{7}$?
6. What is $\frac{5}{7}$ of $\frac{3}{4}$? $\frac{6}{7}$ of $\frac{3}{4}$? $\frac{3}{7}$ of $\frac{14}{5}$?
7. In reducing a compound fraction to a simple one, what do we do with the numerators, and what with the denominators?

WRITTEN EXERCISES.

1. What is the value of $\frac{3}{4}$ of $\frac{5}{6}$?

SOLUTION.—$\frac{1}{4}$ of $\frac{5}{6}$ equals $\frac{5}{24}$ (Prin. 3), and since $\frac{1}{4}$ of $\frac{5}{6}$ equals $\frac{5}{24}$, $\frac{3}{4}$ of $\frac{5}{6}$ equals 3 times $\frac{5}{24}$, which by Prin. 1, equals $\frac{15}{24}$ or $\frac{5}{8}$.

OPERATION.

$\frac{3}{4}$ of $\frac{5}{6}$ = $\frac{3\times5}{4\times6}=\frac{5}{8}$, *Ans.*

Rule.—*Multiply the numerators together and the denominators together, cancelling the factors common to both terms.*

NOTE.—Reduce whole or mixed numbers to fractions before commencing the reduction to a simple fraction. To reduce complex fractions to simple ones, see Art. 183.

What is

2. $\frac{4}{5}$ of $\frac{6}{7}$?	*Ans.* $\frac{24}{35}$.		8. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$?	*Ans.* $\frac{2}{5}$.	
3. $\frac{5}{6}$ of $\frac{3}{4}$?	*Ans.* $\frac{5}{8}$.		9. $\frac{3}{5}$ of $\frac{4}{7}$ of $\frac{10}{12}$?	*Ans.* $\frac{2}{7}$.	
4. $\frac{6}{7}$ of $\frac{9}{10}$?	*Ans.* $\frac{27}{35}$.		10. $\frac{5}{6}$ of $\frac{7}{8}$ of $\frac{24}{35}$?	*Ans.* $\frac{1}{2}$.	
5. $\frac{7}{8}$ of $\frac{12}{14}$?	*Ans.* $\frac{3}{4}$.		11. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{6}{7}$ of $\frac{21}{28}$?	*Ans.* $\frac{12}{35}$.	
6. $\frac{11}{12}$ of $\frac{16}{22}$?	*Ans.* $\frac{2}{3}$.		12. $\frac{4}{9}$ of $\frac{12}{16}$ of $\frac{25}{64}$ of $\frac{96}{75}$?	*Ans.* $\frac{8}{45}$.	
7. $\frac{18}{19}$ of $\frac{19}{18}$?	*Ans.* 1.		13. $\frac{18}{25}$ of $\frac{35}{27}$ of $\frac{77}{88}$ of $\frac{117}{154}$?	*Ans.* $\frac{273}{440}$.	

MENTAL EXERCISES.

1. A man gave his son $\frac{1}{2}$ of $\frac{1}{3}$ of a dollar; how much did the boy receive?

2. A man owning $\frac{1}{2}$ of the stock of a bank, sold $\frac{1}{3}$ of his stock how much did he sell?

3. A boy earned $\frac{1}{2}$ of a dollar and gave his mother $\frac{1}{4}$ of it; how much did he give his mother?

4. Having $\frac{1}{2}$ of a bushel of apples I gave $\frac{1}{3}$ of them to my sister· what part of a bushel did I give her?

5. Having lost $\frac{1}{4}$ of my money I found $\frac{1}{3}$ of what I lost, and then had $140; how much had I at first?

6. At $\frac{1}{2}$ of a dollar a gallon, what will $\frac{1}{3}$ of a gallon of syrup cost?

7. At $\frac{1}{3}$ of a dollar a yard, what will $\frac{1}{4}$ of a yard of cloth cost?

8. At $6$$\frac{1}{2}$ a barrel, what will $\frac{1}{3}$ of a barrel of flour cost?

9. At $8$$\frac{1}{3}$ a case, what will $\frac{1}{4}$ of a case of slates cost?

WRITTEN EXERCISES.

1. A had $\frac{1}{7}$ of a ton of hay, which is $\frac{3}{4}$ as much as B has; how much has B?

SOLUTION.—If $\frac{1}{7}$ of a ton of hay is $\frac{1}{4}$ of what B has, $\frac{1}{4}$ of what B has is $\frac{1}{4}$ of $\frac{1}{7}$, which is $\frac{1}{28}$ of a ton, and $\frac{4}{4}$ of what B has is 4 times $\frac{1}{28}$ of a ton, which is $\frac{4}{28}$ of a ton. Therefore, etc.

2. A has $\frac{5}{7}$ of a certain sum of money, which is $\frac{2}{3}$ of what B has; how much has B? *Ans.* $1\frac{5}{14}$.

3. A barrel of flour cost $8$$\frac{4}{5}$, and a barrel of fish cost $\frac{3}{4}$ as much; what was the cost of the fish? *Ans.* $6$$\frac{3}{5}$.

4. A lady bought $\frac{7}{8}$ of a yard of velvet, at $15$$\frac{3}{5}$ a yard: what did it cost? *Ans.* $13$$\frac{13}{20}$.

5. Henry having $1\frac{2}{5}$ of a quart of nuts, divided them among 8 of his schoolmates; what did each receive? *Ans.* $\frac{1}{10}$.

6. A owns $\frac{4}{7}$ of the stock of a railroad, and $\frac{3}{4}$ of this is $3\frac{1}{7}$ times what B owns; how much does B own? *Ans.* $\frac{1}{5}$.

7. Mary shared $1\frac{3}{5}$ of a bushel of berries with 8 of her schoolmates; what did each receive? *Ans.* $\frac{2}{15}$.

8. I drew $580 from bank, which was $\frac{5}{6}$ of what still remained in bank; what was my bank deposit? *Ans.* $1276.

9. William lost $\frac{2}{3}$ of $\frac{5}{6}$ of his money, and found that $132 was $\frac{3}{4}$ of $\frac{5}{6}$ of $1\frac{1}{2}$ of what remained; how much had he at first? *Ans.* $486.

COMMON DENOMINATOR.

167. A Common Denominator is a denominator common to several fractions, or a denominator to which all may be reduced.

168. Similar Fractional Units are those which are of the same kind; as 3 *fifths* and 2 *fifths*.

169. Dissimilar Fractional Units are those which are of different kinds; as, 3 *fourths* and 3 *fifths*.

Principle.—*A common denominator of several fractions must be a common multiple of their denominators.*

MENTAL EXERCISES.

1. Reduce $\frac{1}{3}$ and $\frac{3}{4}$ to a common denominator.

SOLUTION.—A common denominator for *thirds* and *fourths* is *twelfths*. In *one* there are $\frac{12}{12}$, and in $\frac{1}{3}$ there are $\frac{1}{3}$ of $\frac{12}{12}$, or $\frac{4}{12}$, and in $\frac{3}{4}$, etc.

Reduce to a common denominator,

2. $\frac{2}{3}$ and $\frac{3}{4}$.
3. $\frac{3}{5}$ and $\frac{5}{6}$.
4. $\frac{4}{5}$ and $\frac{5}{8}$.
5. $\frac{4}{7}$ and $\frac{5}{8}$.
6. $\frac{5}{8}$ and $\frac{7}{9}$.
7. $\frac{7}{8}$ and $\frac{9}{10}$.

8. Describe the process of reducing two fractions to a common denominator.

WRITTEN EXERCISES.

1. Reduce $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{7}$ to a common denominator.

SOLUTION.—Since the product of the denominators of the fractions is a common multiple of their denominators, $4\times5\times7$, which equals 140, will be the common denominator. Then multiplying both terms of $\frac{3}{4}$ by 5×7 we have $\frac{3}{4}=\frac{105}{140}$ (Prin. 5). Multiplying both terms of $\frac{4}{5}$ by 4×7, we have $\frac{4}{5}=\frac{112}{140}$, etc. Hence the following

OPERATION.

$\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{7}$ = $\frac{105}{140}$, $\frac{112}{140}$, $\frac{100}{140}$

Rule.—*Multiply both terms of each fraction by the denominators of the other fractions.*

Reduce to a common denominator,

2. $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$. *Ans.* $\frac{144}{192}$, $\frac{160}{192}$, $\frac{168}{192}$.
3. $\frac{2}{5}$, $\frac{4}{7}$, and $\frac{5}{6}$. *Ans.* $\frac{84}{210}$, $\frac{120}{210}$, $\frac{175}{210}$.
4. $\frac{4}{5}$, $\frac{3}{4}$, and $\frac{6}{11}$. *Ans.* $\frac{176}{220}$, $\frac{165}{220}$, $\frac{120}{220}$.
5. $\frac{7}{8}$, $\frac{8}{9}$, and $\frac{10}{11}$. *Ans.* $\frac{693}{792}$, $\frac{704}{792}$, $\frac{720}{792}$.
6. $\frac{3}{5}$, $\frac{5}{8}$, and $1\frac{1}{2}$. *Ans.* $\frac{288}{480}$, $\frac{300}{480}$, $\frac{440}{480}$.
7. $\frac{14}{16}$, $\frac{13}{12}$, and $4\frac{1}{8}$. *Ans.* $\frac{1344}{1536}$, $\frac{1664}{1536}$, $\frac{6336}{1536}$.
8. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$. *Ans.* $\frac{576}{1152}$, $\frac{768}{1152}$, $\frac{864}{1152}$, $\frac{960}{1152}$, $\frac{1008}{1152}$.

9. Show that the common denominator of several fractions is a common multiple of the denominators of those fractions.

LEAST COMMON DENOMINATOR.

170. The **Least Common Denominator** of several fractions is the smallest denominator to which all may be reduced.

Principle.—*The least common denominator of several fractions is the least common multiple of their denominators.*

1. Reduce $\frac{2}{3}$, $\frac{5}{8}$, and $\frac{7}{8}$ to their least common denominator.

SOLUTION.—We find the least common multiple of the denominators to be 24, hence 24 is the least common denominator. Dividing 24 by 3, the denominator of $\frac{2}{3}$, we find we must multiply 3 by 8 to produce 24; hence multiplying both terms of $\frac{2}{3}$ by 8, we have $\frac{2}{3}=\frac{16}{24}$ (Prin. 5). Dividing 24 by 6, the denominator of $\frac{5}{6}$, we find we must multiply 6 by 4 to produce 24; hence, multiplying both terms by 4, we have $\frac{5}{6}=\frac{20}{24}$, etc.

OPERATION.

L. C. M.=24

$\frac{2}{3}=\frac{2\times8}{3\times8}=\frac{16}{24}$

$\frac{5}{6}=\frac{5\times4}{6\times4}=\frac{20}{24}$

$\frac{7}{8}=\frac{7\times3}{8\times3}=\frac{21}{24}$

Rule.—I. *Find the least common multiple of the denominators, for the least common denominator.*

II. *Divide the least common denominator by the denominator of each fraction, and multiply both terms by the quotient.*

NOTE.—Reduce compound fractions to simple ones, mixed numbers to improper fractions, and all to their lowest terms, before finding the least common denominator.

To their least common denominator,

2. Reduce $\frac{5}{6}$, $\frac{4}{5}$, $\frac{7}{12}$. *Ans.* $\frac{30}{36}$, $\frac{16}{36}$, $\frac{21}{36}$.
3. Reduce $\frac{3}{4}$, $\frac{7}{10}$, $\frac{5}{6}$. *Ans.* $\frac{45}{60}$, $\frac{42}{60}$, $\frac{50}{60}$.
4. Reduce $\frac{1}{4}$, $1\frac{1}{2}$, $1\frac{3}{10}$. *Ans.* $\frac{15}{60}$, $\frac{55}{60}$, $\frac{39}{60}$.
5. Reduce $4\frac{1}{2}$, $5\frac{2}{3}$, $7\frac{3}{4}$. *Ans.* $\frac{54}{12}$, $\frac{68}{12}$, $\frac{93}{12}$.
6. Reduce $\frac{3}{5}$, $\frac{17}{20}$, $\frac{19}{24}$. *Ans.* $\frac{72}{120}$, $\frac{102}{120}$, $\frac{95}{120}$.
7. Reduce $\frac{6}{7}$, $\frac{12}{14}$, $\frac{15}{18}$, $\frac{21}{36}$. *Ans.* $\frac{168}{252}$, $\frac{216}{252}$, $\frac{210}{252}$, $\frac{147}{252}$.
8. Reduce $2\frac{3}{7}$, $5\frac{4}{7}$, $\frac{13}{14}$, $\frac{9}{10}$, $\frac{29}{35}$. *Ans.* $\frac{182}{70}$, $\frac{390}{70}$, $\frac{65}{70}$, $\frac{63}{70}$, $\frac{58}{70}$.
9. Reduce $\frac{2}{3}$ of $\frac{7}{8}$, $\frac{4}{5}$ of $3\frac{3}{4}$, $11\frac{13}{51}$. *Ans.* $\frac{119}{204}$, $\frac{612}{204}$, $\frac{2822}{204}$.
10. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$. *Ans.* $\frac{1260}{2520}$, $\frac{840}{2520}$, etc.

ADDITION OF FRACTIONS.

171. **Addition of Fractions** is the process of finding the sum of two or more fractions.

PRINCIPLES.

1 *To add two or more fractions, they must express similar fractional units.*

2. *To add two or more fractions they must be reduced to a common denominator.*

MENTAL EXERCISES.

1. How many sixths in $\frac{1}{2}$ and $\frac{2}{3}$?

SOLUTION.—$\frac{1}{2}$ is equal to $\frac{3}{6}$ and $\frac{2}{3}$ is equal to $\frac{4}{6}$; $\frac{3}{6}$ and $\frac{4}{6}$ are $\frac{7}{6}$, which is equal to $1\frac{1}{6}$.

Find the sum

2. Of $\frac{1}{2}$ and $\frac{1}{4}$.
3. Of $\frac{2}{3}$ and $\frac{1}{4}$.
4. Of $\frac{3}{4}$ and $\frac{1}{5}$.
5. Of $\frac{1}{2}$ and $\frac{2}{5}$.
6. Of $\frac{2}{3}$ and $\frac{1}{5}$.
7. Of $\frac{1}{5}$ and $\frac{1}{7}$.
8. Of $\frac{3}{4}$ and $\frac{1}{7}$.
9. Of $\frac{1}{5}$ and $\frac{1}{7}$.

10. How then shall we add two fractions whose denominators are unlike?

WRITTEN EXERCISES.

1. What is the sum of $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{7}{8}$?

SOLUTION.—Reducing the fractions to a common denominator that they may express similar fractional units, we have $\frac{3}{4}=\frac{18}{24}$, $\frac{5}{8}=\frac{20}{24}$, $\frac{7}{8}=\frac{21}{24}$; 18 *twenty-fourths* plus 20 *twenty-fourths* plus 21 *twenty-fourths* equals 59 *twenty-fourths*. Hence the following

OPERATION.

$$\frac{3}{4}+\frac{5}{8}+\frac{7}{8}=$$
$$\frac{18}{24}+\frac{20}{24}+\frac{21}{24}=\frac{59}{24}.$$

Rule.—*Reduce the fractions to a common denominator, then add the numerators and write the sum over the common denominator.*

NOTES.—1. Reduce compound fractions to simple ones, and reduce each fraction and the sum to lowest terms.

2. To add mixed numbers, add the integers and fractions separately, and then unite their sums.

2. Find the sum of $\frac{4}{5}$, $\frac{6}{8}$, $\frac{9}{12}$. *Ans.* $2\frac{3}{10}$.
3. Find the sum of $\frac{4}{6}$, $\frac{7}{9}$, $\frac{5}{6}$. *Ans.* $2\frac{5}{7\,2}$.
4. Find the sum of $\frac{5}{6}$, $\frac{11}{12}$, $\frac{13}{18}$. *Ans.* $2\frac{17}{36}$.
5. Find the sum of $\frac{4}{7}$, $\frac{3}{4}$, $\frac{11}{14}$. *Ans.* $2\frac{3}{28}$.
6. Find the sum of $2\frac{1}{2}$, $4\frac{3}{4}$, $\frac{11}{12}$. *Ans.* $8\frac{1}{6}$.
7. Find the sum of $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{12}$. *Ans.* $3\frac{3}{8}$.
8. Find the sum of $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{7}{8}$. *Ans.* $3\frac{7}{40}$.
9. Find the sum of $\frac{5}{6}$, $\frac{7}{8}$, $\frac{9}{10}$, $\frac{11}{12}$. *Ans.* $3\frac{31}{40}$.
10. Find the sum of $2\frac{1}{2}$, $4\frac{3}{8}$, $3\frac{1}{4}$, $1\frac{9}{10}$. *Ans.* $12\frac{19}{40}$.
11. Find the sum of $4\frac{3}{4}$, $7\frac{1}{8}$, $9\frac{1}{16}$, $7\frac{1}{50}$. *Ans.* $27\frac{383}{400}$.
12. Find the sum of $21\frac{4}{7}$, $35\frac{1}{11}$, $22\frac{3}{21}$, and $4\frac{1}{2}$. *Ans.* $83\frac{47}{154}$.
13. Find the sum of $17\frac{7}{8}$, $49\frac{8}{15}$, $24\frac{6}{11}$, $18\frac{1}{33}$. *Ans.* $109\frac{433}{440}$.
14. Find the sum of $\frac{3}{4}$ of $\frac{5}{6}$, $\frac{4}{5}$ of $\frac{6}{7}$, $\frac{5}{6}$ of $\frac{7}{8}$. *Ans.* $2\frac{67}{168}$.
15. Find the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$. *Ans.* $1\frac{2841}{2520}$.

SUBTRACTION OF FRACTIONS.

172. Subtraction of Fractions is the process of finding the difference between two fractions.

PRINCIPLES.

1. *To subtract two fractions they must express similar fractional units.*

2. *To subtract two fractions they must be reduced to a common denominator.*

MENTAL EXERCISES.

1. How many 12ths in the difference between $\frac{3}{4}$ and $\frac{2}{3}$?

SOLUTION.—$\frac{3}{4}$ equals $\frac{9}{12}$ and $\frac{2}{3}$ equals $\frac{8}{12}$, and the difference between $\frac{9}{12}$ and $\frac{8}{12}$ is $\frac{1}{12}$.

Subtract

2. $\frac{1}{2}$ from $\frac{1}{3}$.	6. $\frac{2}{3}$ from $\frac{5}{6}$.
3. $\frac{2}{3}$ from $\frac{3}{4}$.	7. $\frac{4}{7}$ from $\frac{4}{5}$.
4. $\frac{3}{4}$ from $\frac{4}{5}$.	8. $\frac{3}{4}$ from $\frac{7}{8}$.
5. $\frac{2}{5}$ from $\frac{1}{2}$.	9. $\frac{5}{6}$ from $\frac{7}{8}$.

10. How then shall we subtract two fractions whose denominators are unlike?

WRITTEN EXERCISES.

1. What is the difference between $\frac{5}{8}$ and $\frac{7}{9}$?

SOLUTION.—Reducing the fractions to a common denominator that they may express similar fractional units, we have $\frac{7}{9}=\frac{63}{72}$ and $\frac{5}{8}=\frac{45}{72}$: 56 *seventy-seconds* minus 45 *seventy-seconds* equals 11 *seventy-seconds*. Hence the following

OPERATION.

$$\frac{7}{9}-\frac{5}{8}=$$
$$\frac{63}{72}-\frac{45}{72}=\frac{11}{72}$$

Rule.—*Reduce the fractions to a common denominator, take the difference of the numerators, and write it over the common denominator.*

NOTE.—Reduce compound fractions to simple ones, and reduce each fraction and the difference to its lowest terms.

Subtract

2. $\frac{4}{5}$ from $\frac{5}{7}$.	*Ans.* $\frac{4}{35}$.	9. $\frac{13}{15}$ from $\frac{14}{19}$.	*Ans.* $\frac{23}{285}$.
3. $\frac{3}{7}$ from $\frac{4}{9}$.	*Ans.* $\frac{10}{63}$.	10. $\frac{14}{26}$ from $\frac{23}{39}$.	*Ans.* $\frac{28}{117}$.
4. $\frac{3}{4}$ from $\frac{7}{9}$.	*Ans.* $\frac{1}{36}$.	11. $\frac{7}{12}$ from $\frac{3}{4}$ of $\frac{2}{10}$.	*Ans.* $\frac{11}{120}$.
5. $\frac{8}{9}$ from $\frac{9}{10}$.	*Ans.* $\frac{1}{90}$.	12. $1\frac{1}{2}$ from $\frac{4}{7}$ of $3\frac{1}{2}$.	*Ans.* $1\frac{17}{42}$.
6. $\frac{10}{11}$ from $\frac{11}{12}$.	*Ans.* $\frac{1}{132}$.	13. $\frac{8}{9}$ from $\frac{3}{5}$ of $\frac{16}{30}$.	*Ans.* $\frac{1}{45}$.
7. $\frac{8}{15}$ from $\frac{13}{14}$.	*Ans.* $\frac{83}{210}$.	14. $\frac{3}{5}$ of $\frac{15}{18}$ from $\frac{7}{8}$ of $\frac{24}{28}$.	*Ans.* $\frac{6}{12}$.
8. $\frac{13}{14}$ from $\frac{17}{18}$.	*Ans.* $\frac{1}{63}$.	15. $\frac{3}{4}$ of $\frac{7}{8}$ from $\frac{4}{5}$ of $\frac{15}{16}$.	*Ans.* $\frac{3}{32}$.

16. Subtract $8\frac{5}{7}$ from $12\frac{3}{7}$.

SOLUTION.—We cannot subtract $\frac{5}{7}$ from $\frac{3}{7}$, so we take 1 from 12, which added to $\frac{3}{7}$ equals $1\frac{3}{7}$ or $\frac{10}{7}$; $\frac{5}{7}$ from $\frac{10}{7}$ leaves $\frac{5}{7}$, and 8 from 11 leaves 3; hence the difference is $3\frac{5}{7}$.

17. Subtract $8\frac{5}{6}$ from $12\frac{1}{6}$. *Ans.* $3\frac{1}{3}$.

18. Subtract $5\frac{7}{8}$ from $10\frac{3}{8}$. *Ans.* $4\frac{1}{2}$.

19. Subtract $10\frac{3}{9}$ from $20\frac{7}{9}$. *Ans.* $9\frac{4}{9}$.

20. Subtract $12\frac{7}{11}$ from $24\frac{8}{11}$. *Ans.* $11\frac{7}{11}$.

21. Subtract $20\frac{1}{12}$ from $30\frac{3}{4}$. *Ans.* $9\frac{5}{6}$.

22. Subtract $40\frac{7}{9}$ from $60\frac{5}{12}$. *Ans.* $19\frac{23}{36}$.

WRITTEN PROBLEMS

IN ADDITION AND SUBTRACTION OF FRACTIONS.

1. A has \$5$\frac{1}{2}$, B has \$4$\frac{1}{4}$, and C has \$6$\frac{3}{4}$; how much money have they all? *Ans.* \$16$\frac{1}{2}$.

2. A miller ground $7\frac{1}{3}$ bushels of corn for A, $9\frac{3}{4}$ for B, $10\frac{1}{6}$ for C; how much did he grind in all? *Ans.* $27\frac{17}{12}$ bu.

3. A lady bought material for a wrapper costing \$$1\frac{1}{10}$, and buttons costing \$$\frac{9}{100}$; what change should she receive from a \$5 bill? *Ans.* \$$3\frac{1}{2}$.

4. A lady went shopping with \$100 and paid \$$12\frac{1}{2}$ for a bonnet, \$$32\frac{3}{4}$ for a dress, and \$$52\frac{5}{8}$ for a cloak; how much money did she bring home? *Ans.* \$2.12$\frac{1}{2}$.

5. A boy gave $12\frac{1}{2}$ cents for a slate, $18\frac{3}{4}$ cents for a knife, $37\frac{1}{2}$ cents for a grammar, and $62\frac{1}{2}$ cents for an arithmetic; what did they all cost? *Ans.* \$1.31$\frac{1}{4}$

6. A merchant bought two pieces of muslin, each containing $41\frac{5}{6}$ yd.; after selling $57\frac{3}{4}$ yd. from them, how many yards remained? *Ans.* $25\frac{1}{2}$ yd.

7. Mr. Weeks finds that his family burned last winter $1\frac{1}{2}$ tons of coal in December, $2\frac{5}{8}$ tons of coal in January, $2\frac{3}{16}$ tons of coal in February, and in March $1\frac{3}{8}$ tons; how much was burned during the four months? *Ans.* $7\frac{11}{16}$ tons.

8. Four loads of hay weighed upon the scales $49\frac{7}{25}$ hundredweight, $43\frac{4}{25}$ hundredweight, $39\frac{1}{20}$ hundredweight, and $45\frac{9}{100}$ hundredweight; what was the weight of the hay, the weight of the wagon being $15\frac{7}{25}$ hundredweight?

Ans. $115\frac{23}{50}$ hundredweight.

MULTIPLICATION OF FRACTIONS.

173. **Multiplication of Fractions** is the process of finding a product when one or both factors are fractions.

174. There are **Three Cases: 1st. A fraction by an integer; 2d. An integer by a fraction; 3d. A fraction by a fraction.**

CASE I.

175. ***To multiply a fraction by an integer.***

MENTAL EXERCISES.

1. How many are 5 times $\frac{2}{15}$?

SOLUTION.—5 times $\frac{2}{15}$ are $\frac{10}{15}$, which reduced to its lowest terms equals $\frac{2}{3}$. Therefore, etc.

SOLUTION 2d.—5 times $\frac{1}{15}$ equal $\frac{5}{15}$, or $\frac{1}{3}$; if 5 times $\frac{1}{15}$ equal $\frac{1}{3}$, 5 times $\frac{2}{15}$ equals 2 times $\frac{1}{3}$ or $\frac{2}{3}$; hence 5 times $\frac{2}{15}$ equals $\frac{2}{3}$.

How many are

2. 8 times $\frac{5}{9}$?
3. 6 times $\frac{7}{18}$?
4. 7 times $\frac{3}{11}$?
5. 8 times $\frac{1}{12}$?
6. 8 times $5\frac{3}{8}$?
7. 4 times $7\frac{1}{4}$?
8. 8 times $6\frac{3}{4}$?
9. 9 times $4\frac{7}{17}$?

10. Since 5 times $\frac{2}{15}$ equals $\frac{2}{3}$, how may this result be obtained by omitting the analysis?

11. How then may a fraction be multiplied by omitting the analysis?

WRITTEN EXERCISES.

1. Multiply $\frac{11}{12}$ by 4

SOLUTION.—Multiplying the numerator (Prin. 1), we have 4 times $\frac{11}{12}$ equals $\frac{44}{12}$; or, dividing the denominator (Prin. 4), we have 4 times $\frac{11}{12}=\frac{11}{3}$, or $3\frac{2}{3}$.

OPERATION.

$\frac{11}{12}\times 4=\frac{44}{12}$

$\frac{11}{12}\times 4=\frac{11}{3}$

Rule.—*To multiply a fraction by an integer, multiply the numerator, or divide the denominator.*

Multiply

2. $\frac{11}{12}$ by 6. *Ans.* $5\frac{1}{2}$.
3. $\frac{14}{15}$ by 5. *Ans.* $4\frac{2}{3}$.
4. $\frac{17}{18}$ by 12. *Ans.* $11\frac{1}{3}$.
5. $\frac{13}{36}$ by 18. *Ans.* $9\frac{1}{2}$.
6. $\frac{15}{28}$ by 14. *Ans.* $7\frac{1}{2}$.
7. $\frac{115}{112}$ by 28. *Ans.* $28\frac{3}{4}$.
8. $\frac{195}{224}$ by 32. *Ans.* 15.
9. $\frac{3239}{17278}$ by 144. *Ans.* $19\frac{1}{12}$.
10. $7\frac{9}{10}$ by 5. *Ans.* $39\frac{1}{2}$.
11. $18\frac{7}{8}$ by 3. *Ans.* $56\frac{1}{8}$.
12. $28\frac{15}{20}$ by 5. *Ans.* $143\frac{3}{4}$.
13. $17\frac{18}{21}$ by 7. *Ans.* 125.
14. $24\frac{19}{24}$ by 12. *Ans.* $297\frac{1}{2}$.
15. $46\frac{15}{26}$ by 13. *Ans* $605\frac{1}{2}$.
16. $128\frac{19}{54}$ by 18. *Ans.* $2310\frac{1}{3}$.
17. $428\frac{18}{121}$ by 11. *Ans.* $4709\frac{7}{11}$.

CASE II.

176. *To multiply an integer by a fraction.*

MENTAL EXERCISES.

1. Multiply 12 by $\frac{3}{4}$.

SOLUTION.—12 multiplied by *one* equals 12, hence 12 multiplied by $\frac{1}{4}$ equals $\frac{1}{4}$ of 12, which is 3, and 12 multiplied by $\frac{3}{4}$ equals 3 times 3, which are 9.

2. 16 by $\frac{1}{4}$.
3. 25 by $\frac{1}{5}$.
4. 35 by $\frac{1}{7}$.
5. 26 by $\frac{1}{8}$.
6. 30 by $\frac{1}{7}$.
7. 40 by $2\frac{1}{4}$.
8. 50 by $3\frac{1}{5}$.
9. 60 by $4\frac{1}{8}$.

10. In multiplying 12 by $\frac{3}{4}$, by what do we divide 12, and by what do we multiply the quotient?

11. How then may we multiply a whole number by a fraction?

WRITTEN EXERCISES.

1. Multiply 7 by $\frac{4}{5}$.

SOLUTION.—7 multiplied by *one* equals 7, hence 7 multiplied by $\frac{1}{5}$ equals $\frac{1}{5}$ of 7, which is $\frac{7}{5}$, and 7 multiplied by $\frac{4}{5}$ equals 4 times $\frac{7}{5}$, which equals $\frac{28}{5}$. Therefore, etc.

OPERATION.

$7 \times \frac{4}{5} = \frac{28}{5}$

2. Multiply 28 by $7\frac{3}{5}$.

SOLUTION.—Multiplying 28 by 3 and dividing by 5, we have $28 \times \frac{3}{5}$ equals $16\frac{4}{5}$; and then multiplying 28 by 7, we have 196; adding 196 to $16\frac{4}{5}$, we have $212\frac{4}{5}$. Therefore, etc.

OPERATION.

$$\begin{array}{r} 28 \\ 7\frac{3}{5} \\ \hline 5)\overline{84} \\ \hline 16\frac{4}{5} \\ 196 \\ \hline 212\frac{4}{5} \end{array}$$ *Ans.*

NOTE.—This method of multiplying by a mixed number is more convenient than the one usually presented.

Rule.—*Multiply the integer by the numerator of the multiplier, and divide the product by the denominator; or divide first and then multiply.*

Multiply

3. 9 by $\frac{2}{3}$. *Ans.* 6.
4. 10 by $\frac{4}{5}$. *Ans.* 8.
5. 18 by $\frac{8}{9}$. *Ans.* 16.
6. 27 by $\frac{5}{7}$. *Ans.* $19\frac{2}{7}$.
7. 28 by $3\frac{4}{7}$. *Ans.* 100.
8. 180 by $15\frac{4}{5}$. *Ans.* 2844.
9. 144 by $\frac{17}{24}$. *Ans.* 102.
10. 284 by $\frac{25}{36}$. *Ans.* $276\frac{1}{9}$.
11. 256 by $7\frac{15}{16}$. *Ans* 2032.
12. 180 by $25\frac{17}{24}$. *Ans.* $4627\frac{1}{2}$.
13. 196 by $\frac{9}{15}$ of $\frac{25}{28}$. *Ans.* $110\frac{1}{4}$.
14. 648 by $\frac{40}{65}$ of $\frac{78}{84}$. *Ans.* $370\frac{2}{7}$.

CASE III.

177. *To multiply a fraction by a fraction.*

MENTAL EXERCISES.

1. What is the product of $\frac{4}{7}$ by $\frac{3}{5}$?

SOLUTION.—$\frac{4}{7}$ multiplied by *one* equals $\frac{4}{7}$, hence $\frac{4}{7}$ multiplied by $\frac{3}{5}$ equals $\frac{3}{5}$ of $\frac{4}{7}$, which is $\frac{12}{35}$, and $\frac{4}{7}$ multiplied by $\frac{3}{5}$ equals 3 times $\frac{4}{35}$, which is $\frac{12}{35}$. Therefore, etc.

What is the product

2. Of $\frac{3}{4}$ by $\frac{2}{3}$?
3. Of $\frac{7}{8}$ by $\frac{12}{11}$?
4. Of $\frac{9}{10}$ by $\frac{5}{18}$?
5. Of $\frac{7}{24}$ by $\frac{3}{10}$?
6. Of $2\frac{1}{2}$ by $3\frac{1}{3}$?
7. Of $7\frac{1}{4}$ by $\frac{2}{3}$?
8. Of $1\frac{1}{10}$ by $1\frac{1}{10}$?
9. Of $5\frac{1}{2}$ by $4\frac{1}{3}$?

10. In multiplying $\frac{4}{7}$ by $\frac{3}{5}$, what do we multiply together for the numerator of the product and what for the denominator?

11. How then do we multiply a fraction by a fraction?

WRITTEN EXERCISES.

1. What is the product of $\frac{5}{6}$ by $\frac{7}{8}$?

SOLUTION.—$\frac{5}{6}$ multiplied by *one* equals $\frac{5}{6}$, hence $\frac{5}{6}$ multiplied by $\frac{1}{8}$ equals $\frac{1}{8}$ of $\frac{5}{6}$, which is $\frac{5}{48}$, and $\frac{5}{6}$ multiplied by $\frac{7}{8}$ equals 7 times $\frac{5}{48}$, which is $\frac{35}{48}$. Therefore, etc.

OPERATION.

$\frac{5}{6} \times \frac{7}{8} = \frac{35}{48}$

SOLUTION 2D.—$\frac{5}{6}$ times $\frac{7}{8}$ equals $\frac{1}{8}$ of 5 times $\frac{7}{6}$; 5 times $\frac{7}{6}$ equals $\frac{35}{6}$ (Prin. 1); and $\frac{1}{8}$ of $\frac{35}{6}$ equals $\frac{35}{48}$ (Prin. 3).

Rule.—*Multiply the numerators together and the denominators together, cancelling common factors.*

NOTE.—Practically this case is the same as finding a fractional part of a fraction, but theoretically the two cases are entirely distinct.

What is the product of

2. $\frac{5}{6}$ by $\frac{3}{7}$? *Ans.* $\frac{5}{14}$.
3. $\frac{10}{12}$ by $\frac{6}{15}$? *Ans.* $\frac{1}{3}$.
4. $\frac{42}{99}$ by $\frac{33}{56}$? *Ans.* $\frac{7}{24}$.
5. $\frac{3}{4} \times \frac{5}{6} \times \frac{6}{7} \times \frac{7}{8}$? *Ans.* $\frac{15}{32}$.
6. $\frac{14}{21} \times \frac{9}{28} \times \frac{35}{54}$? *Ans.* $\frac{10}{63}$.
7. $\frac{9}{10} \times \frac{10}{27} \times \frac{35}{48}$? *Ans.* $\frac{7}{16}$.
8. $\frac{18}{24} \times \frac{28}{45} \times \frac{115}{116}$? *Ans.* $\frac{161}{432}$.
9. $\frac{45}{49} \times \frac{56}{65} \times \frac{91}{72}$? *Ans.* 1.
10. $12\frac{1}{2} \times 8\frac{1}{3}$? *Ans.* $104\frac{1}{6}$.
11. $17\frac{2}{5} \times 16\frac{2}{3}$? *Ans.* 290.
12. $27\frac{3}{11} \times 21\frac{5}{6}$? *Ans.* $595\frac{5}{11}$.
13. $39\frac{3}{5} \times 33\frac{1}{3}$. *Ans.* 1320.
14. Multiply $\frac{9}{10}$ of $\frac{18}{35}$ by $\frac{7}{24}$ of $\frac{14}{27}$. *Ans.* $\frac{3}{25}$.
15. Multiply $\frac{14}{28}$ of $\frac{18}{24}$ by $\frac{28}{45}$ of $\frac{115}{116}$. *Ans.* $\frac{28}{87}$.
16. Multiply $\frac{18}{27}$ of $\frac{102}{95}$ by $\frac{144}{150}$ of $\frac{2728}{17228}$. *Ans.* $\frac{2231}{14430}$.

MENTAL EXERCISES.

1. How much will $4\frac{1}{2}$ yards of muslin cost, at 6 cents a yard?

2. If you give to 8 boys each $2\frac{3}{4}$ apples, how many apples will it take?

3. How many dollars does that man give away who gives to each of 10 beggars $\frac{1}{4}$ of a dollar?

4. How much will 15 inkstands cost at the rate of 2 inkstands for $\frac{1}{4}$ of a dollar?

5. What will 12 Letter-writers cost at the rate of 8 for $\frac{9}{10}$ of a dollar?

6. If a man can dig a rod of ditch in $\frac{1}{2}$ of a day, how many rods can he dig in $4\frac{1}{2}$ days?

7. Multiplying by $\frac{1}{4}$ is the same as taking what part of a number?

8. Show that a fractional part of an integer equals the product of the integer by the fraction.

9. What will be the cost of $2\frac{1}{2}$ yards of cloth at $\$3\frac{1}{4}$ a yard?

10. What will $7\frac{1}{4}$ bushels of shelled corn cost at $25\frac{1}{2}$ cents a bushel?

11. A lady bought $4\frac{3}{4}$ dozen spoons at $\$6\frac{2}{3}$ a dozen; what did they cost her?

WRITTEN EXERCISES.

Required the cost of

1. 96 lb. of paper, at $2\frac{3}{8}$ cts. per pound? *Ans.* \$2.72.
2. 475 lb. of beef, at $6\frac{3}{4}$ cts. per pound? *Ans.* $\$32.06\frac{1}{4}$.
3. 130 yd. of muslin, at $18\frac{3}{4}$ cts. a yard? *Ans.* $\$24.37\frac{1}{2}$.
4. 387 sheep, if 1 sheep cost $8\frac{1}{3}$ dollars? *Ans.* $\$3328\frac{1}{3}$.
5. 180 tons of coal, at $\$10.62\frac{1}{2}$ per ton? *Ans.* $\$1912\frac{1}{2}$.
6. $729\frac{3}{8}$ lb. of sugar, at 6 cts. a pound? *Ans.* \$43.78.
7. $8\frac{1}{7}$ barrels of flour, at $\$6\frac{3}{4}$ a barrel? *Ans.* $\$56\frac{1}{4}$.
8. $6\frac{2}{3}$ yards of cloth, at $\$5\frac{1}{4}$ a yard? *Ans.* $\$33\frac{1}{2}$.
9. $3\frac{4}{7}$ bushels of apples, at $\$\frac{5}{8}$ a bushel? *Ans.* $\$2\frac{9}{14}$.
10. 24 handkerchiefs, at $87\frac{1}{2}$ cents apiece? *Ans.* \$21.
11. $6\frac{4}{7}$ tons of hay, at $\$6\frac{7}{8}$ a ton? *Ans.* $\$46\frac{1}{2}$.
12. $8\frac{3}{4}$ cords of wood, at $\$4\frac{2}{3}$ a cord? *Ans.* $\$38\frac{1}{15}$.
13. 15 barrels of vinegar, at $\$4\frac{1}{4}$ a barrel? *Ans.* $\$71\frac{1}{4}$.
14. $16\frac{1}{2}$ quarts of beans, at $\$0.18\frac{3}{4}$ a quart? *Ans.* $\$3.09\frac{3}{8}$.
15. What is the value of $\frac{24}{25}\times\frac{75}{77}\times\frac{3}{64}\times\frac{242}{243}$? *Ans.* $\frac{11}{168}$.
16. What is the value of $\frac{42}{64}\times\frac{17}{121}\times\frac{144}{289}\times\frac{45}{243}$? *Ans.* $\frac{1}{121}$.
17. Value of $\frac{729}{961}\times\frac{999}{1089}\times\frac{121}{1024}\times\frac{124}{1001}$? *Ans.* $\frac{243}{1232}$.

NOTE.—In examples like 15, 16, 17, and 18, resolve the numerators and denominators into their factors and then cancel.

18. Value of $\frac{289}{441} \times \frac{231}{840} \times \frac{700}{841} \times \frac{870}{999} \times \frac{1998}{985}$? *Ans.* $\frac{10}{29}$.

19. Value of $\overline{(\frac{1}{3}+\frac{8}{15})} \times \frac{3}{11} + (\frac{4}{17}+\frac{5}{6}) \times 3$? *Ans.* $3\frac{827}{1870}$.

20. Value of $(\frac{5}{8}+\frac{10}{11}) \times (\frac{3}{4}+\frac{7}{15}) \times \frac{72}{78} \times \frac{208}{280}$? *Ans.* $3\frac{3}{10}$.

21. Value of $7\frac{7}{10} \times 4\frac{6}{11} + (4\frac{1}{2} - 3\frac{4}{5}) - (5\frac{6}{7}+6\frac{3}{11})$?

Ans. $23\frac{433}{770}$.

22. Find the value of $732\frac{1}{11} - (33\frac{1}{3}+37\frac{1}{2})$ diminished by $(6\frac{1}{4}+6\frac{7}{12}) \times 49\frac{5}{18}$. *Ans.* $20\frac{4155}{1056}$.

23. Mr. Fish rented a house at $\$42\frac{3}{4}$ a month, taking a lease for 5 years; but disposed of the lease at the end of $3\frac{1}{4}$ years; how much rent did he pay? *Ans.* $\$166\frac{7}{4}$.

24. A bill of books at retail amounts to $\$376\frac{3}{8}$, but I get a reduction of $\frac{1}{3}$ for wholesale and $\frac{3}{50}$ for cash; what was the exact amount of the bill? *Ans.* $\$235\frac{217}{800}$.

25. Mr. Alden bought $52\frac{1}{2}$ bushels of wheat at $62\frac{1}{2}$ cts per bushel, and sold $\frac{1}{3}$ at $69\frac{1}{2}$ cts., and the remainder at $71\frac{3}{4}$ cts. per bushel; what did he clear by the transaction?

Ans. $\$4.43\frac{1}{8}$.

DIVISION OF FRACTIONS.

178. Division of Fractions is the process of dividing when one or both of the terms are fractional.

179. There are **Three Cases**: 1st. A fraction by an integer; 2d. An integer by a fraction; 3d. A fraction by a fraction.

CASE I.

180. ***To divide a fraction by an integer.***

MENTAL EXERCISES.

1. How many times is 4 contained in $\frac{8}{11}$?

SOLUTION.—*One* is contained in $\frac{8}{11}$, $\frac{8}{11}$ times; and if 1 is contained in $\frac{8}{11}$, $\frac{8}{11}$ times, 4 is contained in $\frac{8}{11}$, $\frac{1}{4}$ of $\frac{8}{11}$ times, or $\frac{2}{11}$ times. Therefore, etc.

How many times is

2. 3 contained in $\frac{7}{8}$?
3. 5 contained in $1\frac{5}{6}$?
4. 6 contained in $\frac{8}{9}$?
5. 7 contained in $1\frac{3}{13}$?
6. 4 contained in $8\frac{1}{2}$?
7. 6 contained in $12\frac{3}{5}$?
8. 7 contained in $6\frac{3}{8}$?
9. 9 contained in $17\frac{1}{10}$?

10. Since $\frac{8}{11} \div 4 = \frac{2}{11}$, how may we divide a fraction by a number that will divide the numerator?

11. Since $\frac{10}{11} \div 4 = \frac{10}{44}$, how may we divide a fraction by a number that will not divide the numerator?

WRITTEN EXERCISES.

1. Divide $\frac{10}{11}$ by 5, and also by 6.

SOLUTION.—Dividing the numerator of $\frac{10}{11}$ by 5, we have $\frac{10}{11} \div 5$ equals $\frac{2}{11}$ (Prin. 2).

OPERATION.
$\frac{10}{11} \div 5 = \frac{2}{11}$

SOLUTION 2D.—Multiplying the denominator of $\frac{10}{11}$ by 6, we have $\frac{10}{11} \div 6$ equals $\frac{10}{66}$, or $\frac{5}{33}$ (Prin. 3).

OPERATION.
$\frac{10}{11} \div 6 = \frac{10}{66} = \frac{5}{33}$.

2. Divide $822\frac{5}{6}$ by 4.

SOLUTION.—Dividing 822 by 4, we have 205 and a remainder of 2; 2 equals $\frac{12}{6}$, which, added to $\frac{5}{6}$, equals $\frac{17}{6}$; $\frac{17}{6}$ divided by 4 equals $\frac{17}{24}$; hence the quotient is $205\frac{17}{24}$.

OPERATION.
$4)822\frac{5}{6}$
$205\frac{17}{24}$

Rule.—*Divide the numerator or multiply the denominator of the dividend by the divisor.*

NOTE.—Reduce a mixed number to a fraction; or divide the integer, unite the remainder with the fraction, and divide the result.

Divide

3. $\frac{2}{10}$ by 3.	*Ans.* $\frac{2}{10}$.	**10.** $785\frac{5}{6}$ by 7.	*Ans.* $112\frac{3}{35}$.
4. $\frac{16}{15}$ by 4.	*Ans.* $\frac{4}{15}$.	**11.** $287\frac{5}{8}$ by 12.	*Ans.* $23\frac{7}{32}$.
5. $\frac{18}{25}$ by 36.	*Ans.* $\frac{1}{50}$.	**12.** $129\frac{3}{5}$ by 16.	*Ans.* $8\frac{1}{10}$.
6. $\frac{3}{4}$ of $\frac{14}{9}$ by 7.	*Ans.* $\frac{1}{6}$.	**13.** $878\frac{4}{11}$ by 15.	*Ans.* $58\frac{22}{165}$.
7. $\frac{5}{6}$ of $\frac{72}{125}$ by 9.	*Ans.* $\frac{4}{75}$.	**14.** $1284\frac{2}{13}$ by 26.	*Ans.* $49\frac{18}{338}$.
8. $\frac{7}{8}$ of $\frac{72}{91}$ by 18.	*Ans.* $\frac{1}{26}$.	**15.** $1347\frac{1}{17}$ by 30.	*Ans.* $44\frac{6}{51}$.
9. $2\frac{4}{21}$ by $\frac{7}{8}$ of $3\frac{3}{7}$.	*Ans.* $\frac{46}{63}$.	**16.** $2892\frac{7}{20}$ by 36.	*Ans.* $80\frac{247}{720}$

CASE II.

181. ***To divide an integer by a fraction.***

MENTAL EXERCISES.

1. How many times is $\frac{1}{4}$ contained in 3?

SOLUTION.—*One* is contained in 3, 3 times; and if 1 is contained in 3, 3 times, $\frac{1}{4}$ is contained in 3, 4 times 3 times, which are 12 times; and $\frac{3}{4}$ is contained in 3, $\frac{1}{4}$ of 12 times, or 4 times. Therefore, etc.

How many times is

2. $\frac{1}{3}$ contained in 6?	**6.** $1\frac{1}{2}$ contained in 3?
3. $\frac{1}{8}$ contained in 8?	**7.** $1\frac{3}{4}$ contained in 7?
4. $\frac{7}{8}$ contained in 7?	**8.** $2\frac{3}{10}$ contained in 46?
5. $\frac{4}{5}$ contained in 11?	**9.** $1\frac{3}{5}$ contained in 30?

10. In dividing 12 by $\frac{3}{4}$, by what do we multiply 12, and by what do we divide the product?

11. How then may we divide a whole number by a fraction without the analysis?

WRITTEN EXERCISES.

1. Divide 10 by the fraction $\frac{5}{6}$.

SOLUTION.—10 divided by *one* equals 10, hence 10 divided by $\frac{1}{6}$ equals 6 times 10, and 10 divided by 5 sixths equals $\frac{1}{5}$ of 6 times 10, which is $\frac{6}{5}$ times 10, which equals 12. Hence the

OPERATION

$10 \div \frac{5}{6} =$
$10 \times \frac{6}{5} = 12$ *Ans.*

Rule I.—*Multiply the dividend by the denominator of the fraction, and divide the product by the numerator.*

Rule II.—*Invert the divisor and proceed as in multiplication of fractions.*

Divide

2. 5 by $\frac{3}{5}$.	*Ans.* $8\frac{1}{3}$.	8. 15 by $8\frac{3}{4}$.	*Ans.* $1\frac{5}{7}$.
3. 9 by $\frac{4}{7}$.	*Ans.* $15\frac{3}{4}$.	9. 256 by $3\frac{7}{8}\frac{7}{5}$.	*Ans.* 80.
4. 12 by $\frac{6}{7}$.	*Ans.* 14.	10. 552 by $2\frac{1}{2}\frac{1}{9}$.	*Ans.* 232.
5. 27 by $2\frac{1}{4}$.	*Ans.* 12.	11. 558 by $8\frac{8}{7}$.	*Ans.* 63.
6. 30 by $\frac{5}{8}$.	*Ans.* 48.	12. 638 by $19\frac{1}{3}$.	*Ans.* 33.
7. 144 by $2\frac{2}{5}$.	*Ans.* 54.	13. 729 by $22\frac{1}{2}$.	*Ans.* $32\frac{2}{5}$.

CASE III.

182. ***To divide a fraction by a fraction.***

MENTAL EXERCISES.

1. How many times is $\frac{1}{4}$ contained in $\frac{3}{5}$?

SOLUTION.—*One* is contained in $\frac{5}{5}$, $\frac{5}{3}$ times, and if 1 is contained in $\frac{5}{3}$, $\frac{5}{3}$ times, $\frac{1}{4}$ is contained in $\frac{5}{3}$, 4 times $\frac{5}{3}$ times, which is $\frac{20}{3}$, or $6\frac{2}{3}$ times; and $\frac{3}{4}$ is contained in $\frac{3}{5}$, $\frac{1}{5}$ of $\frac{1}{4}$, or $\frac{1}{20}$ times. Therefore, etc.

How many times is

2. $\frac{1}{3}$ contained in $\frac{1}{7}$?	6. $\frac{3}{4}$ contained in $1\frac{1}{2}$?
3. $\frac{1}{3}$ contained in $\frac{4}{7}$?	7. $\frac{5}{6}$ contained in $2\frac{1}{2}$?
4. $\frac{5}{8}$ contained in $\frac{5}{6}$?	8. $\frac{7}{11}$ contained in $8\frac{8}{11}$?
5. $\frac{7}{11}$ contained in $1\frac{1}{3}$?	9. $\frac{4}{5}$ contained in $9\frac{3}{5}$?

10. In dividing $\frac{5}{6}$ by $\frac{3}{4}$ we multiply $\frac{5}{6}$ by what, and divide the product by what?

11. How then may we divide a fraction by a fraction without the analysis?

WRITTEN EXERCISES.

1. Divide $\frac{7}{8}$ by $\frac{5}{6}$.

SOLUTION.—$\frac{7}{8}$ divided by *one* equals $\frac{7}{1}$, hence $\frac{7}{8}$ divided by $\frac{1}{6}$ equals 6 times $\frac{7}{8}$, and $\frac{7}{8}$ divided by 5 sixths equals $\frac{1}{5}$ of 6 times $\frac{7}{8}$, which is $\frac{6}{5}$ times $\frac{7}{8}$, which equals $\frac{4}{4}\frac{2}{0}$, or $\frac{2}{2}\frac{1}{0}$. Therefore we have the following

OPERATION.

$\frac{7}{8} \div \frac{5}{6} =$
$\frac{7}{8} \times \frac{6}{5} = \frac{4}{4}\frac{2}{0}$ or
$= \frac{2}{2}\frac{1}{0}$. *Ans.*

Rule I.—*Multiply the dividend by the denominator of the divisor, and divide by the numerator.*

Rule II.—*Invert the divisor and proceed as in multiplication of fractions.*

NOTE.—Reduce mixed numbers to simple fractions. When the divisor is a compound fraction, invert each term and multiply, cancelling when possible.

Divide

2. $\frac{7}{8}$ by $\frac{3}{4}$. *Ans.* $1\frac{1}{6}$.
3. $\frac{9}{10}$ by $\frac{4}{5}$. *Ans.* $1\frac{1}{8}$.
4. $\frac{37}{52}$ by $\frac{3}{65}$. *Ans.* $1\frac{5}{16}$.
5. $\frac{14}{25}$ by $1\frac{3}{15}$. *Ans.* $\frac{3}{10}$.
6. $\frac{48}{85}$ by $\frac{72}{105}$. *Ans.* 2.
7. $\frac{84}{99}$ by $\frac{48}{77}$. *Ans.* $1\frac{13}{36}$.
8. $\frac{45}{112}$ by $\frac{15}{56}$. *Ans.* $1\frac{1}{2}$.
9. $\frac{105}{144}$ by $\frac{245}{432}$. *Ans.* $1\frac{7}{7}$.
10. $\frac{105}{121}$ by $\frac{175}{198}$. *Ans.* $\frac{54}{55}$.
11. $7\frac{5}{24}$ by $12\frac{8}{15}$. *Ans.* $\frac{865}{1604}$.
12. $15\frac{15}{16}$ by $8\frac{6}{11}$. *Ans.* $1\frac{889}{1936}$.
13. $17\frac{7}{19}$ by $10\frac{4}{5}$. *Ans.* $1\frac{585}{855}$.
14. $21\frac{3}{7}$ by $12\frac{8}{21}$. *Ans.* $1\frac{19}{26}$.
15. $25\frac{4}{5}$ by $15\frac{4}{11}$. *Ans.* $1\frac{209}{1521}$.
16. $33\frac{1}{3}$ by $12\frac{1}{25}$. *Ans.* $2\frac{624}{1001}$.
17. $\frac{9}{16}$ by $\frac{9}{20}$ of $\frac{8}{39}$. *Ans.* $1\frac{17}{48}$.
18. $\frac{28}{42}$ of $7\frac{9}{10}$ by $\frac{72}{84}$. *Ans.* $6\frac{18}{60}$.
19. $\frac{45}{80} \div \frac{90}{112}$ by $\frac{27}{40} \times \frac{112}{135}$. *Ans.* $1\frac{1}{4}$.

MENTAL EXERCISES.

1. If a yard of cloth cost $\frac{1}{8}$ of a dollar, how many yards can you buy for 12 dollars?

2. Mary gave $12\frac{1}{2}$ for silk at the rate of $2\frac{1}{2}$ a yard; how many yards did she buy?

3. How long will it require for a man to earn $33, at the rate of $2\frac{1}{4}$ a day?

4. If $2\frac{1}{2}$ barrels of apples cost $15, how many barrels can be bought for 12 dollars?

5. How many building lots of $\frac{1}{4}$ of an acre each, can we make out of 12 acres?

6. A lady distributed $29 equally among some poor people, giving them $5\frac{1}{4}$ apiece; how many did she aid?

7. A school girl bought 6 yards of ribbon worth $5\frac{1}{4}$ cents a yard; how many apples worth $1\frac{1}{2}$ cents each would cost the same?

8. Mrs. Bear exchanged 20 pounds of butter, at 15 cents a pound, for calico, worth $12\frac{1}{2}$ cents a yard; how many yards did she receive?

9. A school-boy shared 14 apples equally with his companions, giving to each $3\frac{1}{2}$ apples; required the number of his companions.

10. A grocer sold 8 bushels of potatoes, worth $\$\frac{1}{3}$ a bushel, and received for them eggs at the rate of $\$\frac{1}{3}$ a dozen; how many eggs did he get?

WRITTEN EXERCISES.

1. If a pound of brown sugar cost $7\frac{7}{10}$ cents, how many pounds can you buy for $25\frac{4}{5}$ cents? *Ans.* $3\frac{1}{3}$.

SUGGESTION.—The analysis gives $\frac{254}{10}\times\frac{10}{77}$, which by cancelling and multiplying equals $3\frac{1}{3}$.

2. If a quart of vinegar cost $18\frac{3}{4}$ cents, how many quarts can be bought for $52\frac{1}{2}$ cents? *Ans.* $2\frac{4}{5}$.

3. If 7 yards of book muslin cost $\$5\frac{87}{100}$, how much will 5 yards cost? *Ans.* \$4.10.

4. If $15\frac{3}{5}$ yards of ribbon cost 45 cents, what will 6 yards cost? *Ans.* $17\frac{4}{13}$ cts.

5. If $21\frac{3}{4}$ pounds of tea cost $\$18\frac{38}{100}$, what will 1 pound cost? *Ans.* \$0.85.

6. If $22\frac{1}{5}$ barrels of sugar cost $\$308\frac{1}{40}$, what is the price per barrel? *Ans.* $\$13\frac{7}{8}$.

7. If $28\frac{1}{4}$ tons of Lehigh coal cost $\$183\frac{5}{8}$, what is the price per ton? *Ans.* $\$6\frac{1}{2}$.

8. Divide $\frac{4}{5}$ of $\frac{7}{12}$ of $\frac{9}{10}$ by $\frac{6}{8}$ of $\frac{16}{64}$ of $\frac{21}{40}$ of $5\frac{1}{3}$. *Ans.* $\frac{48}{125}$.

9. Find the value of $\frac{33}{72}\times\frac{21\frac{3}{5}}{42\frac{1}{3}}\times\frac{75}{117}\div\frac{33\frac{1}{3}}{26\frac{1}{3}}$. *Ans.* $\frac{11}{108}$.

10. Find the value of $(\frac{33}{81}\times\frac{45}{88})\div(\frac{44}{54}\times\frac{44}{33})$ *Ans.* $\frac{49}{77}$.

11. Find the value of $(\frac{24}{5}-\frac{15}{11})\times\frac{77}{121}$, and the product divided by $(6\frac{3}{4}-5\frac{3}{4})$. *Ans.* $3\frac{31}{55}$.

12. Find the value of $(\frac{55}{63}+3\frac{4}{7})\times9\frac{7}{8}$ and the product divided by $(6\frac{2}{3}-4\frac{1}{12})$. *Ans.* $16\frac{14}{21}$.

13. Find the value of $(77\frac{22}{11}-44\frac{3}{7})\times(6\frac{3}{8}-5\frac{1}{4})$ divided by $(8\frac{3}{4}+7\frac{1}{4}-3\frac{7}{12})$. *Ans.* $2\frac{7}{8}$.

14. If an errand boy earn $\$7\frac{7}{8}$ in a week, how long will it require him to earn $\$20\frac{1}{4}$? *Ans.* $2\frac{4}{7}$ weeks.

15. A lady distributed \$231 among the poor, giving $\$28\frac{7}{8}$ to each person; how many were there? *Ans.* 8 persons.

16. Mr. B divided $\$416\frac{1}{4}$ equally among five persons; how much did each receive? *Ans.* $\$83\frac{1}{4}$.

17. By his father's will, Henry shared \$9600 equally with his five brothers; how much did he receive? *Ans.* \$1600.

18. The product of two numbers, diminished by $112\frac{3}{4}$, is $127\frac{1}{8}$, and one number is 15; required the other.

Ans. $16\frac{1}{3}\frac{1}{6}$.

19. A boy shared 102 apples with his companions, giving to each $6\frac{3}{8}$ apples; required the number of companions.

Ans. 15.

20. A man divided $30\frac{4}{5}$ pounds of flour among the poor, giving to each $2\frac{4}{5}$ pounds; how many persons were there?

Ans. 11 persons.

21. A seamstress bought a sewing-machine for \$85.50, and paid \$40 down; how much must she save each month, to pay for it in 7 months? *Ans.* \$6 50.

22. A man's wages are $\$3\frac{4}{5}$ a day and his daily expenses are $\$1\frac{7}{8}$; how many days must he labor to enable him to buy a suit of clothes worth $\$46\frac{1}{2}$? *Ans.* 24 days.

23. Mr. Landis gained \$228 on the sale of $6\frac{7}{8}$ acres of land; how much more would he have gained on each acre, if he had gained \$382 on the whole? *Ans.* \$22.40.

REDUCTION OF COMPLEX FRACTIONS.

183. The **Reduction of Complex Fractions** is the process of changing them to simple fractions.

NOTE.—A complex fraction is not really a fraction, according to the definition of a fraction. It is rather a complex fractional expression of one fraction divided by another.

1. Reduce $\frac{\frac{3}{4}}{\frac{5}{6}}$ to a simple fraction.

SOLUTION.—This expression means that $\frac{3}{4}$ is to be divided by $\frac{5}{6}$, and inverting the divisor and multiplying, we have $\frac{3}{4}\times\frac{6}{5}$, which equals $\frac{9}{10}$.

OPERATION.

$$\frac{\frac{3}{4}}{\frac{5}{6}}=\frac{3}{4}\div\frac{5}{6}=\frac{3}{4}\times\frac{6}{5}=\frac{9}{10}$$

SOLUTION 2D.—Multiplying both terms of the complex fraction by 12, the least common multiple of the denominators of the terms, and reducing the resulting fraction to its lowest terms, we have $\frac{9}{10}$.

OPERATION.

$$\frac{\frac{3}{4}}{\frac{5}{6}}=\frac{\frac{3}{4}\times 12}{\frac{5}{6}\times 12}=\frac{9}{10}, \textit{Ans.}$$

Rule I.—*Multiply the numerator of the complex fraction by its denominator inverted.*

Rule II.—*Multiply both terms of the complex fraction by the least common multiple of the denominators.*

WRITTEN EXERCISES.

Reduce to simple fractions

2. $\frac{\frac{5}{6}}{\frac{7}{8}}$. *Ans.* $\frac{20}{21}$.

3. $\frac{\frac{7}{8}}{\frac{11}{12}}$. *Ans.* $\frac{21}{22}$.

4. $\frac{\frac{14}{15}}{\frac{21}{25}}$. *Ans.* $2\frac{2}{3}$.

5. $\frac{\frac{9}{10}\times\frac{25}{27}}{\frac{16}{16}\times\frac{17}{18}}$. *Ans.* $\frac{16}{17}$.

6. $\frac{\frac{10}{18}\times\frac{33}{45}}{\frac{16}{25}\times\frac{45}{44}}$. *Ans.* $1\frac{13}{27}$.

7. $\frac{\frac{17}{35}\times\frac{25}{85}}{\frac{19}{25}\times\frac{105}{152}}$. *Ans.* $1\frac{33}{532}$.

8. $\frac{2\frac{1}{2}}{3\frac{3}{8}}$. *Ans.* $\frac{16}{27}$.

9. $\frac{6\frac{6}{8}}{9\frac{7}{11}}$. *Ans.* $\frac{11}{16}$.

10. $\frac{10\frac{10}{13}}{12\frac{1}{4}}$. *Ans.* $\frac{80}{91}$.

11. $\frac{\frac{1}{5}+\frac{3}{7}}{\frac{4}{9}+\frac{2}{5}}$. *Ans.* $\frac{99}{134}$.

12. $\frac{3\frac{1}{2}-2\frac{1}{3}}{4\frac{1}{4}-2\frac{1}{5}}$. *Ans.* $\frac{70}{123}$.

13. $\frac{12\frac{1}{2}}{41\frac{2}{3}}\times\frac{3\frac{7}{8}}{6\frac{1}{5}}$. *Ans.* $\frac{3}{16}$.

RELATION OF NUMBERS.

184. The **Relation of Numbers** is their relative value as compared with one another.

NOTE.—This subject is equivalent to Ratio, but is presented here as affording an excellent illustration of the analysis of numbers. The treatment of the subject under Ratio is *demonstrative* rather than *analytic*.

CASE I.

185. *To find the relation of an integer to an integer.*

1. 27 is how many times 6, or what is the relation of 27 to 6?

SOLUTION.—*One* is $\frac{1}{6}$ of 6, and if 1 is $\frac{1}{6}$ of 6, 27 is 27 times one-sixth of 6, which is $\frac{27}{6}$, or $\frac{9}{2}$, or $4\frac{1}{2}$ times 6. Therefore 27 is $4\frac{1}{2}$ times 6. Hence we have the following

Rule.—*Divide the number which you compare by the number with which it is compared.*

NOTE.—The rule is the same for each case, and need not be repeated.

MENTAL EXERCISES.

1. 9 is how many times 6? 25 is how many times 10? 64 is how many times 36?

2. 16 is what part of 36? 18 is what part of 40? 32 is what part of 72? 90 is what part of 108?

3. A watch cost $40, and a chain cost $12; what part of the cost of the watch equals the cost of the chain?

4. If 5 yards of cloth cost $21, what will 20 yards cost at the same rate?

5. If 6 tons of coal are bought for $19½, what will 30 tons cost at the same rate?

6. A has $20 and B has ⅘ as much, plus $3; what part of A's money equals B's money?

WRITTEN EXERCISES.

What part is

2. 25 of 75?	*Ans.* ⅓.	6. 105 of 225?	*Ans.* $\frac{7}{15}$.
3. 18 of 54?	*Ans.* ⅓.	7. 420 of 588?	*Ans.* ⅗.
4. 76 of 24?	*Ans.* $\frac{19}{6}$.	8. 540 of 432?	*Ans.* ⅝.
5. 84 of 91?	*Ans.* $\frac{12}{13}$.	9. 1899 of 2743?	*Ans.* $\frac{9}{13}$.

10. If 16 apples cost 13 cents, what will 48 apples cost at the same rate?

SOLUTION.—If 16 apples cost 13 cents, 48 apples, which are $\frac{48}{16}$, or 3 times 16 apples, cost 3 times 13 cents, or 39 cents.

OPERATION.
16 A. cost 13 cts.
48 A. cost 3×13 cts.
= 39 cts.

11. If 15 oranges cost 14 cents, what will 60 oranges cost at the same rate? *Ans.* 56 cents.

12. What cost 125 sheep, if 25 sheep cost 137 dollars?
Ans. $685.

13. If 56 cows cost $1000, what will 42 cows cost at the same rate? *Ans.* $750.

14. If 105 pens cost 77 cents, what will 225 pens cost at the same rate? *Ans.* $1.65.

NOTE.—Classes needing a shorter course may omit the other three cases unless otherwise directed.

CASE II.

186. ***To find the relation of a fraction to a number.***

1. The fraction ⅔ is what part of 6?

SOLUTION.—*One* is ⅙ of 6, and if 1 is ⅙ of 6, ¼ is ¼ of ⅙, which is $\frac{1}{24}$ of 6, and ⅔ is 3 times $\frac{1}{24}$, which is ⅛ of 6.

MENTAL EXERCISES.

1. What part of 3 is ⅔? Of 4 is ⅔? Of 7 is ⅔? Of 9 is ⅔?

2. What part of 5 is ½ of ¼? Of 7 is ½ of ¼? Of 5 is ½ of ⅛?

3. If ⅓ of a ton of coal cost $2½, what will 6 tons cost at the same rate?

4. A hat was bought for \$2 and sold at a gain of \$$\frac{1}{4}$; what part of the cost equals the gain?

5. Sarah bought 5 yards of silk and used $3\frac{1}{4}$ yards; what part of the silk then remained?

6. A bought 6 oranges, and B bought 4 oranges; what did B pay for his oranges, if A paid 26 cents for his?

WRITTEN EXERCISES.

What is the relation of

2. $\frac{3}{4}$ to 9? *Ans.* $\frac{1}{12}$.
3. $\frac{8}{7}$ to 12? *Ans.* $\frac{1}{28}$.
4. $\frac{7}{10}$ to 14? *Ans.* $\frac{1}{20}$.
5. $\frac{14}{15}$ to 28? *Ans.* $\frac{1}{30}$.
6. $\frac{8}{9}$ to 64? *Ans.* $\frac{1}{72}$.
7. $\frac{8}{35}$ to 35? *Ans.* $\frac{1}{35}$.
8. $\frac{56}{57}$ to 14? *Ans.* $\frac{4}{57}$.
9. $\frac{21}{16}$ to 35? *Ans.* $\frac{3}{80}$.

10. A having \$25, gave \$5 to B and $\frac{1}{4}$ of the remainder to C: what part of \$25 then remained? *Ans.* $\frac{3}{5}$.

CASE III.

187. ***To find the relation of a number to a fraction.***

1. 6 is how many times $\frac{3}{4}$?

SOLUTION.—$\frac{1}{4}$ is $\frac{1}{3}$ of $\frac{3}{4}$, and since $\frac{1}{4}$ is $\frac{1}{3}$ of $\frac{3}{4}$, $\frac{4}{4}$ or 1 is 4 times $\frac{1}{3}$, or $\frac{4}{3}$ of $\frac{3}{4}$, and 6 is 6 times $\frac{4}{3}$, or $\frac{24}{3}$, which equals 8.

MENTAL EXERCISES.

1. 8 is how many times $\frac{4}{5}$? 14 is how many times $\frac{7}{8}$? 24 is how many times $\frac{6}{7}$?

2. 96 is how many times $\frac{16}{17}$? 115 is how many times $\frac{23}{36}$? 144 is how many times $\frac{12}{11}$?

3. If a yard of cloth cost \$$\frac{1}{4}$, how many yards will \$9 buy?

4. A can walk a certain distance in $2\frac{1}{2}$ hours; how many times that distance can he walk in 6 hours?

5. A book cost \$4 and a slate \$$\frac{1}{4}$; how many times the cost of the slate equals the cost of the book?

WRITTEN EXERCISES.

What is the relation of

2. 288 to $\frac{16}{25}$? *Ans.* 414.
3. 495 to $4\frac{2}{5}$? *Ans.* $112\frac{1}{2}$.
4. 1896 to $\frac{24}{27}$? *Ans.* 2133.
5. 462 to $8\frac{5}{6}$? *Ans.* 54.
6. 588 to $3\frac{9}{11}$? *Ans.* 154.
7. 63504 to $50\frac{2}{5}$? *Ans.* 1260

8. Mary had 12 quarts of nuts and Sarah had $\frac{5}{6}$ as many, minus $\frac{3}{4}$ of a quart; how many times Sarah's number equals Mary's? *Ans.* $1\frac{21}{30}$.

CASE IV.

188. ***To find the relation of a fraction to a fraction.***

1. What part of $\frac{14}{15}$ is $\frac{7}{12}$?

SOLUTION.—$\frac{1}{15}$ is $\frac{1}{14}$ of $\frac{14}{15}$, and $\frac{15}{15}$, or *one*, is 15 times $\frac{1}{14}$, which equals $\frac{15}{14}$ of $\frac{14}{15}$. If 1 equals $\frac{15}{14}$ of $\frac{14}{15}$, $\frac{1}{12}$ equals $\frac{1}{12}$ of $\frac{15}{14}$, and $\frac{7}{12}$ equals 7 times $\frac{1}{12}$ of $\frac{15}{14}$, which equals $\frac{7}{12}$ of $\frac{15}{14}$, or $\frac{5}{8}$.

MENTAL EXERCISES.

1. What part of $\frac{2}{3}$ is $\frac{1}{3}$? Of $\frac{1}{3}$ is $\frac{2}{3}$? Of $\frac{5}{7}$ is $\frac{3}{7}$? Of $\frac{3}{7}$ is $\frac{5}{7}$?

2. What part of $\frac{5}{9}$ is $2\frac{1}{2}$? Of $\frac{5}{6}$ is $\frac{3}{4}$ of $\frac{5}{7}$? Of $\frac{3}{5}$ is $\frac{3}{4}$ of $\frac{4}{5}$? Of $\frac{9}{10}$ is $\frac{4}{5}$ of $\frac{7}{8}$?

3. A had $\$\frac{3}{4}$ and spent $\$\frac{2}{5}$; what part of his money did he spend?

4. If $2\frac{1}{2}$ yards of silk cost $\$7\frac{1}{2}$, what will $7\frac{1}{2}$ yards cost at the same rate?

5. A merchant bought $\frac{5}{6}$ of a cargo of oranges and sold $\frac{4}{5}$ of them; what part of the cargo did he sell?

6. Edwin and Emma each had $\$2$; Edwin spent $\frac{1}{4}$ of his, and Emma spent $\frac{1}{5}$ of hers; what part of Emma's money then equaled Edwin's?

WRITTEN EXERCISES.

What is the relation of

2. $\frac{5}{7}$ to $\frac{10}{7}$? *Ans.* $\frac{1}{2}$.
3. $\frac{12}{20}$ to $\frac{57}{40}$? *Ans.* $\frac{2}{3}$.
4. $\frac{17}{21}$ to $\frac{85}{42}$? *Ans.* $\frac{2}{5}$.
5. $\frac{28}{37}$ to $\frac{54}{74}$? *Ans.* $\frac{28}{27}$.
6. $9\frac{11}{15}$ to $11\frac{2}{3}$? *Ans.* $\frac{146}{175}$.
7. $11\frac{12}{13}$ to $7\frac{8}{7}$? *Ans.* $\frac{217}{143}$.
8. $\frac{2}{3}\times\frac{9}{16}$ to $\frac{8}{7}\times\frac{12}{20}$? *Ans.* $\frac{105}{304}$.
9. $\frac{4}{5}\times\frac{16}{18}$ to $\frac{12}{20}\times\frac{24}{27}$? *Ans.* $\frac{16}{9}$.

10. If a merchant sold $\frac{4}{5}$ of $\frac{5}{6}$ of his stock, how many fifths of $\frac{5}{6}$ of his stock remained? *Ans.* $\frac{2}{5}$.

REMARK.—He sold $\frac{4}{5}$, and hence there remained $\frac{1}{5}$, and $\frac{1}{5}$ equals *two fifths* of $\frac{1}{5}$ of his stock; hence the answer is $\frac{2}{5}$.

11. Harry lost $\frac{3}{7}$ of $\frac{7}{9}$ of a certain sum of money; how many sevenths of $\frac{7}{9}$ of the sum remained? *Ans.* $\frac{6}{7}$.

12. Two boys had each $\frac{9}{10}$ of a bushel of walnuts, when the first sold $\frac{2}{3}$ of his to the second; what part of what the second has equals what the first has? *Ans.* $\frac{1}{5}$.

13. James had $\frac{4}{5}$ of a ton of coal and Thomas had $\frac{5}{6}$ of a ton; if James burns $\frac{1}{3}$ of his coal and Thomas $\frac{1}{4}$ of his how many ninths of a ton will remain? *Ans.* $6\frac{19}{20}$ ninths.

NOTE.—For *Greatest Common Divisor* and *Least Common Multiple of Fractions* see *Manual*.

ARITHMETICAL ANALYSIS.

189. Analysis is the process of solving problems by a comparison of their elements. In comparing, we reason to the unit and from the unit; the unit being the basis of the reasoning process.

CASE I.

190. ***To pass from one integer to another integer.***

1. If 5 oranges cost $12\frac{1}{2}$ cents, what will 8 oranges cost at the same rate?

SOLUTION.—If 5 oranges cost $12\frac{1}{2}$ or $\frac{25}{2}$ cents, 1 orange cost $\frac{1}{5}$ of $\frac{25}{2}$, which is $\frac{5}{2}$ cents; and if 1 orange cost $\frac{5}{2}$ cents, 8 oranges will cost 8 times $\frac{5}{2}$ cents, or 20 cents.

OPERATION.

5 cost $\frac{25}{2}$
1 " $\frac{5}{2}$
8 " $\frac{5}{2}\times 8=20$, *Ans.*

MENTAL EXERCISES.

1. If 4 apples cost $5\frac{1}{2}$ cents, what will 5 apples cost at the same rate?
2. If 3 girls earn $\$2\frac{1}{4}$ in a day, how much could 5 girls earn at the same rate?
3. How long will it take a boy to plow 8 furrows, if it takes him $13\frac{1}{4}$ minutes to plow 5 furrows?
4. How much will 9 barrels of flour cost, if 2 barrels cost $\$10\frac{1}{2}$?
5. If 5 hats can be bought for $\$11\frac{1}{4}$, how much will 8 hats cost?
6. What cost 5 pounds of butter at the rate of $\$3\frac{1}{2}$ for 10 pounds?

WRITTEN EXERCISES.

2. If 6 hens cost \$3.54, what will 7 hens cost at the same rate? *Ans.* \$4.13.
3. If 8 turkeys cost $\$19\frac{1}{2}$, what will 10 turkey cost at the same rate? *Ans.* \$24.
4. How much must I pay for 22 yards of muslin, at the rate of 5 yards for $\$\frac{19}{20}$? *Ans.* \$4.18.
5. How much will 21 books cost at the rate of 9 books for $15\frac{3}{7}$? *Ans.* \$36.
6. What must I pay for 8 arithmetics, at the rate of 5 arithmetics for \$4.75? *Ans.* \$7.60.
7. If 3 tons of hay cost $\$47\frac{1}{3}$, how much will 12 tons cost at the same rate? *Ans.* $\$189\frac{1}{3}$.
8. Required the cost of 21 barrels of flour, at the rate of \$17.50 for 3 barrels. *Ans.* \$122.50

9. A gave $43.50 for pigs, at the rate of $7.25 for 3 pigs; how many did he buy? *Ans.* 18 pigs.

10. B gave $54 for flour, at the rate of $6 for $4\frac{1}{2}$ bushels; how many bushels did he buy? *Ans.* 39 bushels.

CASE II.

191. ***To pass from a fraction to an integer.***

1. If $\frac{3}{4}$ of an acre of land cost $108, what will one acre cost?

SOLUTION.—If $\frac{3}{4}$ of an acre cost $108, $\frac{1}{4}$ of an acre cost $\frac{1}{3}$ of $108, which is $36; and if $\frac{1}{4}$ of an acre cost $36, $\frac{4}{4}$ of an acre, or one acre, will cost 4 times $36, which are $144.

OPERATION.

$\frac{3}{4}$ cost $108
$\frac{1}{4}$ " $36
$\frac{4}{4}$ " $144, *Ans.*

MENTAL EXERCISES.

1. If $\frac{4}{5}$ of a yard of cloth is worth 24 dimes, what are 3 yards of the cloth worth?

2. If $\frac{3}{4}$ of a barrel of sugar cost $12, what will 7 barrels cost?

3. I bought 5 yards of summer silk at the rate of $10 for $3\frac{1}{2}$ yards · what was the cost?

4. Mary bought 9 apples, at the rate of $\frac{3}{4}$ of an apple for $\frac{2}{3}$ of a cent; what did they cost?

5. Henry bought 25 peaches, at the rate of $\frac{3}{4}$ of a peach for $\frac{4}{5}$ of a cent; required the cost.

6. Three-fifths of what I paid for a cow was $24; what did I pay for a horse whose value was 5 times that of the cow?

WRITTEN EXERCISES.

2. If $\frac{3}{8}$ of the cost of A's farm is $2480, what did the farm cost? *Ans.* $3720.

3. If $\frac{4}{7}$ of a barrel of flour cost $2.50, what will 5 barrels cost at the same rate? *Ans.* $21.87\frac{1}{2}$.

4. If $\frac{5}{7}$ of a yard of cloth cost $2.10, what will enough for suit containing 9 yards cost? *Ans.* $22.05.

5. If $\frac{5}{8}$ of a cwt. of sugar cost $12.45, what will be the cost of 20 cwt., or 1 ton of sugar? *Ans.* $298.80.

6. What will 21 tons of hay cost, if $\frac{7}{8}$ of a ton of hay cost $12.85? *Ans.* $308.40.

7. If $46\frac{7}{8}$ is $\frac{5}{16}$ of the price I pay for a horse, what would be the cost of 4 pairs of horses at the same rate?

Ans. $1200.

8. If a lawyer's clerk could write 60 pages in $10\frac{2}{3}$ hours, how much would he write in a week, working 10 hours a day? *Ans.* $337\frac{1}{2}$ pages.

9. How much will it cost to transport 25 hundredweight of freight from Philadelphia to Pittsburgh at \$4 for $2\frac{2}{11}$ hundredweight? *Ans.* $\$45\frac{5}{8}$.

10. How much will I pay for 15 acres of land, at the rate of $2\frac{1}{2}$ acres for \$425.25? *Ans.* \$2551.50.

11. A bought 40 bushels of grain, at the rate of $3\frac{1}{8}$ bushels for \$3.20; what did it cost? *Ans.* \$38.40.

12. B sold 77 gallons of milk, at the rate of \$1.25 for $5\frac{1}{2}$ gallons; required the cost. *Ans.* \$17.50.

13. C bought 39 cords of wood, at the rate of \$12.45 for $3\frac{1}{4}$ cords; how much did it cost? *Ans.* \$149.40.

14. D bought 81 tons of coal, at the rate of $5\frac{2}{5}$ tons for \$26.25; how much did it cost? *Ans.* \$393.75.

CASE III.

192. ***To pass from an integer to a fraction.***

1. If 1 barrel of apples cost \$8, what will $\frac{3}{4}$ of a barrel cost?

SOLUTION.—If 1 barrel of apples cost \$8, $\frac{1}{4}$ of a barrel cost $\frac{1}{4}$ of \$8, which is \$2; and if $\frac{1}{4}$ of a barrel cost \$2, $\frac{3}{4}$ of a barrel will cost 3 times \$2, which are \$6.

OPERATION.

1 cost \$8
$\frac{1}{4}$ " \$2
$\frac{3}{4}$ " \$6, *Ans.*

MENTAL EXERCISES.

1. What cost $\frac{3}{8}$ of a pound of beef, if 2 pounds cost 32 cents?

2. If a man can earn $\$5\frac{1}{4}$ in 3 days, how much will he earn in $\frac{1}{3}$ of a day?

3. If 2 barrels of flour cost $\$10\frac{3}{4}$, for what can I sell $\frac{3}{4}$ of a barrel at the rate of cost?

4. If Harry walks $6\frac{3}{4}$ miles in 2 hours, how far will he walk at this rate in $3\frac{1}{2}$ hours?

5. If 7 pounds of sugar cost 56 cents, for what can I sell $4\frac{1}{2}$ pounds, and neither gain nor lose?

6. A vessel sailed 60 miles in 12 hours; how far at this rate did it sail in $4\frac{1}{2}$ hours?

7. If a seamstress earns $\$1\frac{1}{2}$ in 2 days, how much will she earn in $10\frac{1}{2}$ days?

WRITTEN EXERCISES.

2. Henry paid \$24.60 for a cow; how much would he have paid had he given $\frac{4}{5}$ as much? *Ans.* \$19.68.

3. If 3 yards of cloth cost \$7.50, what will $\frac{7}{8}$ of a yard of the same cloth cost? *Ans.* \$2.18$\frac{3}{4}$.

4. If 7 barrels of apples cost \$19$\frac{3}{5}$, what will $\frac{5}{8}$ of a barrel cost at the same rate? *Ans.* \$2$\frac{1}{8}$.

5. How much will 10$\frac{7}{8}$ tons of coal cost at the rate of \$13$\frac{5}{7}$ for 8 tons? *Ans.* \$18$\frac{9}{14}$.

6. A bought 9$\frac{3}{5}$ acres of land, at the rate of \$1605 for 5 acres; required the cost. *Ans.* \$3174$\frac{1}{5}$.

7. How much must I pay for 18$\frac{15}{16}$ cords of wood, at the rate of \$6$\frac{2}{3}$ for 3 cords? *Ans.* \$42$\frac{1}{12}$.

8. If 9 tons of hay are worth \$100$\frac{4}{5}$, what must I give for 14$\frac{3}{14}$ tons? *Ans.* \$167$\frac{1}{5}$.

9. If 12 bushels of beans cost \$11$\frac{1}{4}$, how much will 15$\frac{14}{16}$ bushels cost? *Ans.* \$14$\frac{15}{16}$.

10. How much must I pay for 18$\frac{16}{18}$ tons of iron ore, at the rate of 17 tons for \$33$\frac{3}{4}$? *Ans.* \$37$\frac{1}{2}$.

11. A gave \$1968 for 21 acres of land; for what would he sell $\frac{1}{12}$ of an acre, at a gain of \$4 an acre? *Ans.* \$92$\frac{4}{7}$.

12. B gave \$444 for 24 tons of hay; he sold C $\frac{12}{20}$ of a ton. at \$2$\frac{1}{2}$ less a ton; what did he receive? *Ans.* \$15.20.

NOTE.—The solution of several of the above problems is simplified by *expressing the operations* and *cancelling*.

CASE IV.

193. ***To pass from a fraction to a fraction.***

1. If $\frac{3}{5}$ of an acre of land cost \$150, what will $\frac{4}{5}$ of an acre cost?

SOLUTION.—If $\frac{3}{5}$ of an acre cost \$150, $\frac{1}{5}$ of an acre cost $\frac{1}{3}$ of \$150, which is \$75; and $\frac{4}{5}$ of an acre will cost 3 times \$75, which are \$225. If 1 acre cost \$225, $\frac{1}{5}$ of an acre cost $\frac{1}{3}$ of \$225, which is \$45, and $\frac{4}{5}$ of an acre will cost 4 times \$45, which are \$180.

SOLUTION 2D.—If $\frac{3}{5}$ of an acre cost \$150, $\frac{1}{3}$ of an acre cost $\frac{1}{3}$ of \$150, and $\frac{4}{5}$ of an acre will cost $\frac{4}{3}\times$\$150; and $\frac{4}{5}$ of an acre will cost $\frac{4}{3}\times\frac{1}{3}\times$\$150, which by *cancellation* we find is \$180.

OPERATION.

$\frac{4}{3}\times\frac{1}{3}\times\$150=\$180$, *Ans.*

NOTE.—The 2d method, by cancellation, is preferred, especially with the more difficult problems.

MENTAL EXERCISES.

1. If $\frac{3}{4}$ of an apple cost $\frac{9}{10}$ of a cent, what will $\frac{1}{4}$ of an apple cost?

2. If $\frac{1}{2}$ of a barrel of flour is worth $6\frac{1}{2}$, what is $\frac{1}{7}$ of a barrel worth?

3. What is $\frac{1}{3}$ of an acre of land worth, if $\frac{1}{7}$ of an acre is worth $126?

4. How much must I pay for $\frac{1}{11}$ of a cord of wood, if $\frac{1}{7}$ of a cord is worth $1.50?

5. What must I pay for $\frac{1}{3}$ of a bushel of berries, if $\frac{1}{4}$ of a bushel is worth $6?

6. What will $\frac{1}{7}$ of a yard of cloth cost, if $\frac{1}{8}$ of a yard cost 7\frac{1}{2}$?

7. I paid $1400 for $4\frac{3}{4}$ acres of land; how much will $5\frac{1}{4}$ acres cost at the same rate?

8. A bought $7\frac{1}{2}$ yards of cloth for 10\frac{1}{2}$; what would he have paid for $8\frac{1}{4}$ yards at the same rate?

9. How many apples can you get for $53\frac{1}{2}$ cents, if $2\frac{1}{2}$ apples are worth $6\frac{3}{4}$ cents?

WRITTEN EXERCISES.

2. When $189\frac{1}{2}$ are paid for $8\frac{3}{5}$ acres of land, how much must be paid for $10\frac{3}{5}$ acres? *Ans.* $238\frac{1}{8}$.

SOLUTION.—$189\frac{1}{2} = \frac{379}{2}$; $8\frac{3}{5} = \frac{43}{5}$; $10\frac{3}{5} = \frac{53}{5}$; the cost, we find by analysis, is $\frac{379}{2} \times \frac{5}{43} \times \frac{53}{5} = 238\frac{1}{8}$; reducing by cancellation.

3. A gave $6\frac{5}{7}$ yards of cloth for $13\frac{3}{7}$ cords of wood; how many cords could be got for $9\frac{3}{7}$ yards? *Ans.* $19\frac{1}{8}$ cords.

4. B gave $12\frac{1}{2}$ tons of iron for $32\frac{1}{16}$ barrels of flour; how many barrels would he get for $14\frac{3}{4}$ tons? *Ans.* $38\frac{1}{2}$.

5. C gave $5\frac{3}{8}$ tons of hay for $28\frac{3}{8}$ tons of coal; how much coal would he get for $7\frac{3}{4}$ tons of hay? *Ans.* $41\frac{1}{8}$ tons.

6. If 6 horses eat $7\frac{1}{8}$ bushels of oats in a day, how many horses would eat $33\frac{3}{8}$ bushels in a day? *Ans.* 28.

7. If 9 pigs cost $12\frac{3}{8}$, how many pigs can you buy for $115\frac{1}{2}$ dollars? *Ans.* 84.

8. If $2\frac{1}{2}$ barrels of flour cost $24\frac{3}{8}$, how many barrels can you buy for $87\frac{3}{4}$? *Ans.* 9.

9. How much must I pay for $6\frac{3}{8}$ pounds of sugar, at the rate of $0.44\frac{4}{5}$ for $2\frac{7}{7}$ pounds? *Ans.* $1.28\frac{1}{2}$.

10. How much must I pay for $10\frac{5}{7}$ bushels of rice at the rate of $15\frac{3}{8}$ for $5\frac{3}{7}$ bushels? *Ans* $30\frac{3}{4}$.

11. If $19\frac{3}{5}$ tons of iron cost $42\frac{3}{7}$, how many tons can you buy for $357\frac{1}{7}$? *Ans.* $163\frac{1}{8}$ tons.

12. How many bushels of wheat can I buy for $114\frac{3}{4}$, at the rate of $29\frac{1}{4}$ bushels for $28\frac{1}{8}$? *Ans.* $118\frac{3}{4}$ bushels.

13. If $35\frac{1}{2}$ yards of cloth cost $39\frac{1}{2}$, how much will $5\frac{1}{2}$ yards cost at the same rate? *Ans.* $6.50.

14. How much will $57\frac{1}{4}$ tons of hay cost, at the rate of $41\frac{1}{8}$ tons for $502\frac{1}{2}$? *Ans.* $693.

SUPPLEMENTARY PROBLEMS.

To be omitted unless otherwise directed.

15. What must I pay for $14\frac{1}{2}\frac{1}{7}$ yards of cloth, if $47\frac{3}{11}$ yards cost $378\frac{1}{2}$? *Ans.* $117\frac{1}{2}$.

16. What must I pay for $17\frac{1}{4}$ tons of hay, if $5\frac{1}{8}$ tons cost $70\frac{7}{8}$? *Ans.* $215\frac{1}{2}\frac{1}{0}$.

17. What must I pay for $10\frac{1}{3}$ barrels of rice, if $7\frac{1}{3}$ barrels cost $72\frac{1}{2}$? *Ans.* $103\frac{1}{2}$.

18. If $24\frac{1}{2}$ will buy $25\frac{1}{3}$ bushels of corn, how many dollars will buy $50\frac{1}{9}$ bushels? *Ans.* $48\frac{1}{2}$.

19. If $582\frac{1}{2}$ will buy $13\frac{1}{7}$ tons of iron, how many tons will $1416\frac{1}{2}$ buy? *Ans.* $33\frac{1}{2}$.

20. If $26\frac{3}{11}$ bu. of wheat are given for $6\frac{3}{15}$ tons of hay, how many bushels must be given for $46\frac{1}{2}$ tons of hay? *Ans.* 189 bu.

21. If $7\frac{1}{2}$ yards of cloth are given for $12\frac{2}{5}$ bushels of corn, how many bushels of corn must be given for $19\frac{1}{11}$ yards of cloth? *Ans.* 32 bushels.

CASE V.

194. ***Given a part and the remainder to find the whole.***

1. A man sold $\frac{3}{7}$ of his farm, and then had 216 acres remaining; how many acres in the entire farm?

SOLUTION.—If he sold $\frac{3}{7}$ of his farm, there remained $\frac{7}{7}-\frac{3}{7}$, which is $\frac{4}{7}$ of the farm, which equals 216 acres; if $\frac{4}{7}$ of the farm equals 216 acres, $\frac{1}{7}$ of the farm equals $\frac{1}{4}$ of 216 acres, which is 54 acres, and $\frac{7}{7}$ of the farm, etc.

OPERATION.

$\frac{7}{7}-\frac{3}{7}=\frac{4}{7}$

$\frac{4}{7}=216$

$\frac{1}{7}=54$

$\frac{7}{7}=378$, *Ans.*

MENTAL EXERCISES.

1. A merchant drew out of bank $\frac{1}{4}$ of his deposit, and then had $360 in bank; what was his deposit?

2. A man willed $\frac{1}{8}$ of his property to his wife, and the remainder, $4800, to his son; what was his property?

3. Mary gave $\frac{1}{3}$ of her money for a silk dress and $\frac{1}{4}$ for a cloak, and had $10 remaining; how much had she at first?

4. Frank, after spending $\frac{2}{5}$ of his money, found that $16 was $\frac{1}{3}$ of what he had remaining; how much money had he at first?

5. If $\frac{3}{5}$ of an army were killed, $\frac{1}{6}$ taken prisoners, and 800 men escaped, of how many men did the army consist?

6. What number is that which being increased by its $\frac{1}{2}$, and that sum diminished by $\frac{2}{3}$ of the number, the remainder is 50?

7. When E was married he was 27 years old, and $\frac{2}{3}$ of his age was 4 years more than $\frac{3}{4}$ of his wife's age; required the age of his wife.

8. A boy, after spending $\frac{1}{4}$ of his money for candies and $\frac{1}{5}$ for peaches, found that 20 cents was $\frac{1}{7}$ of what remained; how much money had he?

9. A thief stole $\frac{3}{5}$ of Harry's money, and before he was caught spent $\frac{3}{4}$ of it; the remainder, which was $20 less than he stole, was given back; how much money had Harry at first?

WRITTEN EXERCISES.

2. A invested $\frac{3}{4}$ of his money in bank stock, and had $6045 remaining; how much money had he? *Ans.* $24,180.

3. A drover sold $\frac{2}{9}$ of his cows, and then had 210 remaining; how many had he before the sale? *Ans.* 270.

4. B sold $\frac{1}{3}$ of his gas stock to C and $\frac{1}{5}$ to D, and had remaining 1463 shares; how many shares did he sell to each, and how many had he at first? *Ans.* 3135.

5. A gave $\frac{3}{4}$ of his fortune for a farm, and paid $\frac{3}{5}$ of the remainder for building a house, and then had $4671 remaining; how much had he at first? *Ans.* $46,710.

SUPPLEMENTARY PROBLEMS.

To be omitted unless otherwise directed.

6. A father gave $\frac{1}{2}$ of his fortune to his daughter, $\frac{1}{3}$ to his wife, and the remainder, which was $6720, to his son; what did the wife and the daughter receive? *Ans.* W., $5376; D., $4032.

7. One-third of the trees in an orchard bear apples, $\frac{1}{4}$ bear peaches, and the remainder, which is 100, bear plums; required the number of trees in the orchard. *Ans.* 240.

8. A boy lost $\frac{3}{5}$ of his kite string, and then added $53\frac{1}{4}$ feet, when he found that it was just $\frac{3}{4}$ of its original length; what was its original length? *Ans.* 129 feet.

MENTAL EXERCISES.

MISCELLANEOUS EXAMPLES.

1. If a barrel of apples cost $5\frac{1}{4}$, how many "quarters" does it cost?

2. If 4 boys can earn $6\frac{3}{4}$ in a week, how much does each boy earn?

3. If I buy at the mint 125 "quarters" for change, how many dollars must I pay?

4. What cost 8 barrels of apples, at the rate of $3\frac{3}{4}$ per barrel?

5. $\frac{3}{5}$ of $40 is 2 times what A gave for a bureau; what was the cost of the bureau?

6. $\frac{5}{7}$ of $\frac{14}{15}$ of a dollar is $\frac{1}{10}$ of the cost of a watch; how much did the watch cost?

7. A man owning $\frac{3}{5}$ of the stock of a bank sold $\frac{2}{3}$ of his share; how much did he sell?

8. Jane bought a slate for $\frac{1}{4}$ of a dollar, and an arithmetic for $\frac{5}{8}$ of a dollar; what was the cost of both?

9. If $\frac{1}{2}$ of a number, increased by $\frac{1}{3}$ of the number, equals 50, what is the number?

10. Maria owes a store bill of $\$\frac{3}{5}$; if she hands the clerk $\$\frac{3}{4}$, how much change would she receive?

11. If $\frac{1}{3}$ of a pole is in the air, $\frac{1}{2}$ in the water, and the rest in the mud, how much is in the mud?

12. How much will $4\frac{3}{8}$ yards of muslin cost, at the rate of 12 cents a yard?

13. How much will 10 pounds of sugar cost, at $\frac{4}{5}$ of a dime a pound?

14. If a yard of flannel costs $\$\frac{3}{4}$, how many yards can you buy for $12?

15. Mrs. Brown exchanged 10 pounds of butter, at 15 cents a pound, for calico worth $6\frac{1}{4}$ cents a yard; how many yards did she receive?

16. How much will 14 pounds of cloves cost, if $3\frac{1}{2}$ pounds can be bought for 50 cents?

17. If $2\frac{1}{7}$ yards of muslin cost 30 cents, how much will 10 yards cost, at the same rate?

18. How far will a man drive in $5\frac{1}{2}$ hours, at the rate of 21 miles in $3\frac{1}{2}$ hours?

19. If $\frac{2}{3}$ of a ton of hay is worth $\$4\frac{1}{5}$, what cost $7\frac{3}{4}$ tons of hay at the same rate?

20. John found 60 cents, which is $\frac{2}{3}$ of $\frac{1}{2}$ of what he then had; how much had he at first?

21. Henry gave his sister 20 cents, which is $\frac{1}{3}$ of what he had at first, and $\frac{1}{2}$ of what his sister now has; how much had each at first?

SUPPLEMENTARY MENTAL EXERCISES.

To be omitted unless otherwise directed.

22. Amanda, having 50 pins, lost $\frac{1}{5}$ of them, and then found $\frac{1}{3}$ as many as remained; how many had she then?

23. A gave B 48 cents, and $\frac{1}{5}$ of this is 4 times as much as he had remaining; how much had he at first?

24. If 4 horses eat 2 tons of hay in 8 weeks, how long will it require 5 horses to eat the same?

25. Fanny's number of roses, increased by $\frac{1}{2}$ and $\frac{1}{3}$ of her number, equals 55; how many roses has she?

26. A gave $24 for a watch, and $\frac{1}{2} + \frac{1}{3}$ of this is 4 times what he paid for the chain; required the cost of the chain.

27. A boy having 36 marbles, lost $\frac{2}{3}$ of them, and then found $\frac{1}{2}$ as many as he had at first; how many had he then?

28. $60 is $\frac{2}{3}$ of what B gave for a horse, and the cost of the horse, increased by its $\frac{1}{5}$, is 5 times what he paid for a sleigh; required the cost of the sleigh.

29. A has 40 fruit trees, $\frac{1}{10}$ of which bear apples, $\frac{1}{3}$ of the remainder bear pears, and the rest bear peaches; how many trees of each kind has he?

30. Janson's age, diminished by its $\frac{1}{4}$ and $\frac{1}{5}$, is 22 years, and his age is $\frac{2}{3}$ of his uncle's age; required the age of each.

31. Annie, having $\frac{2}{3}$ of a pound of candies, shared them equally with 5 of her schoolmates; what part of a pound did each receive?

32. Peter gave $\frac{1}{4}$ of his marbles to Samuel, and 2 times $\frac{1}{4}$ of them to Anson; how many had he at first, if he gave away 14 marbles?

33. A boy divided 14 apples equally among his companions, giving to each $3\frac{1}{2}$ apples; required the number of his companions.

34. Harry bought 8 bushels of potatoes, worth $\$\frac{2}{3}$ a bushel, and paid for them with eggs, worth $\$\frac{1}{3}$ a dozen; how many eggs did it take?

35. Hannah's wedding dress cost $50, and $\frac{2}{3}$ of this is 4 times the cost of her bonnet, and also $\frac{2}{3}$ of the cost of her cloak; required the cost of the bonnet and cloak respectively.

36. Mary gave $\frac{1}{4}$ of her money to the poor, and then found $\frac{1}{3}$ as much as she gave away, and then had $30; how much had she at first?

37. William borrowed $\frac{2}{3}$ of Emily's money, and after spending $\frac{1}{4}$ of it, returned the remainder, which was $20; how much money had Emily?

38. A kite in the air fell $\frac{1}{4}$ of the distance to the ground, then arose $\frac{1}{4}$ of the distance it was from the ground, and then fell $\frac{1}{4}$ of the distance it arose; what part of the whole distance was it from the ground?

WRITTEN EXERCISES.

MISCELLANEOUS PROBLEMS.

1. Reduce $45\frac{7}{8}$ to an improper fraction. *Ans.* $\frac{367}{8}$.
2. Reduce $385\frac{9}{13}$ to an improper fraction. *Ans.* $\frac{5014}{13}$.
3. Reduce $\frac{2072}{12}$ to a mixed number. *Ans.* $173\frac{1}{3}$.
4. Reduce $\frac{4785}{125}$ to a mixed number. *Ans.* $38\frac{7}{25}$.
5. Reduce $\frac{655520}{1029960}$ to its lowest terms. *Ans.* $\frac{7}{11}$.
6. Reduce $\frac{2317}{3940}$ to its lowest terms. *Ans.* $\frac{1}{2}\frac{1}{0}$.
7. Reduce $\frac{4}{5}$ of $\frac{15}{16}$ of $\frac{14}{15}$ of $\frac{33}{35}$ to a simple fraction. *Ans.* $\frac{11}{20}$.
8. Reduce $\frac{14}{15}$ of $\frac{18}{25}$ of $\frac{20}{21}$ of $\frac{24}{36}$ of $\frac{63}{88}$ to a simple fraction. *Ans.* $\frac{432}{1575}$.
9. Reduce $\frac{3}{8}$, $\frac{8}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{9}{10}$ to a common denominator.
Ans. $\frac{80}{120}$, $\frac{90}{120}$, $\frac{100}{120}$, $\frac{105}{120}$, $\frac{108}{120}$.
10. Reduce $\frac{4}{5}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{12}$, $\frac{13}{14}$, $\frac{15}{16}$ to a common denominator.
Ans. $\frac{1344}{1680}$, $\frac{1400}{1680}$, $\frac{1470}{1680}$, $\frac{1540}{1680}$, $\frac{1560}{1680}$, $\frac{1575}{1680}$.
11. Reduce $\frac{12}{20}$ of $\frac{17}{19}$ of $\frac{26}{49}$ of $\frac{72}{55}$ of $\frac{57}{85}$ of $\frac{88}{72}$ to a simple fraction. *Ans.* $\frac{8}{85}$.
12. What is the sum of $\frac{4}{5}$, $\frac{11}{14}$, $\frac{9}{16}$, and $\frac{13}{28}$? *Ans.* $2\frac{49}{80}$.
13. Add $\frac{7}{18}$, $\frac{11}{15}$, $\frac{15}{22}$, $\frac{18}{20}$, and $\frac{17}{30}$. *Ans.* $3\frac{41}{198}$.
14. Subtract $\frac{7}{8}$ of $3\frac{3}{7}$ from $\frac{3}{4}$ of $\frac{16}{21}$ of $9\frac{3}{4}$. *Ans.* $2\frac{1}{7}$.
15. Multiply $\frac{3}{4}$ of $3\frac{3}{7}$ by $\frac{3}{4}$ of $\frac{7}{8}$ of $\frac{16}{21}$ of $\frac{18}{27}$. *Ans.* $1\frac{1}{21}$.
16. Divide $\frac{8}{9}$ of $\frac{27}{40}$ of $5\frac{1}{3}$ by $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{16}{12}$. *Ans.* $15\frac{8}{50}$.
17. Add $\frac{8\frac{4}{9}}{16\frac{8}{9}}$ to $\frac{1}{2}$ of $\frac{4}{5}$. *Ans.* $\frac{9}{10}$.
18. Add $\frac{3\frac{1}{2}}{7}$, $\frac{4\frac{1}{3}}{17\frac{1}{3}}$, and $\frac{10}{26\frac{1}{4}}$. *Ans.* $1\frac{11}{34}$
19. Subtract $\frac{45}{270}$ from $\frac{8\frac{1}{3}}{25}$. *Ans.* $\frac{1}{6}$.
20. From 1000 take $\frac{1}{1000}$. *Ans.* $999\frac{999}{1000}$.
21. Multiply $\frac{2}{7}$ of $\frac{21}{88}$ by $\frac{5}{8}$ of $15\frac{\frac{4}{5}}{16}$. *Ans.* $1\frac{296}{608}$.
22. Multiply $\frac{\frac{1}{4}-\frac{1}{9}}{\frac{1}{2}+\frac{1}{3}}+\frac{1}{2}$ by $12\frac{1}{7}$. *Ans.* $8\frac{2}{11}$.
23. Divide $3\frac{3}{8}$ by $\frac{1}{2}$ of $\frac{9\frac{1}{2}}{4}$. *Ans.* $8\frac{5}{57}$.
24. Divide $\frac{3}{4}$ of $\frac{8}{9}$ of $3\frac{1}{12}$ by $\frac{1}{7}$ of $\frac{14}{15}$ of $\frac{10}{27}$ *Ans.* $41\frac{5}{8}$.

25. What cost $45\frac{3}{4}$ yards of muslin at $12\frac{1}{2}$ cents a yard?
Ans. $\$5.71\frac{7}{8}$.

26. What cost $96\frac{7}{8}$ tons of coal, at \$12 per ton?
Ans. $\$1162\frac{1}{2}$.

27. What cost $196\frac{5}{8}$ pounds of meat, at $6\frac{3}{4}$ cents a pound?
Ans. $\$13.27\frac{7}{32}$.

28. What cost $327\frac{3}{8}$ yards of cloth, at $62\frac{1}{2}$ cents a yard?
Ans. $\$204.79\frac{1}{8}$.

29. What cost $\frac{5}{8}$ of a barrel of flour, if 1 barrel cost \$8?
Ans. $\$6\frac{2}{3}$.

30. What cost $\frac{7}{8}$ of a yard of cloth, if 2 yards cost $\$18\frac{2}{3}$?
Ans. \$8.05.

31. What cost $2\frac{3}{5}$ tons of hay, if $3\frac{1}{5}$ tons cost \$48?
Ans. \$40.

32. If 1 yard of cloth cost $\$1\frac{1}{8}$, what will $5\frac{7}{8}$ yards cost?
Ans. \$6.50.

33. If $2\frac{1}{2}$ tons of hay cost \$30, what will $57\frac{3}{4}$ tons cost?
Ans \$693.

34. If 1 yard of cloth cost $\$8.01\frac{3}{7}$, what will $14\frac{4}{7}$ yards cost?
Ans. $\$117\frac{3}{7}$.

35. A has $\frac{3}{4}$ of \$8560, which is $2\frac{1}{2}$ times B's money; how much money has B?
Ans. \$2568.

36. B has 725 acres of land; $2\frac{3}{5}$ times B's equals $3\frac{5}{7}$ times C's; how much has C?
Ans. $507\frac{1}{2}$ acres.

37. If 4 be added to both terms of the fraction $\frac{5}{8}$, will the value of the fraction be increased or diminished?

38. If 4 be subtracted from both terms of the fraction $\frac{5}{8}$, will the value of the fraction be increased or diminished?

39. Will the value of the fraction $\frac{5}{8}$ be increased or diminished if 4 be added to both terms? If 4 be subtracted from both terms?

40. Mr. Mann bought a tract of land for \$6500, and sold $\frac{3}{5}$ of it to Mr. Chase, and $\frac{5}{7}$ of the remainder to Mr. Colburn; what was the cost of the remainder?
Ans. $\$742\frac{6}{7}$.

41. A man worked $25\frac{3}{4}$ days, and after paying his board and other expenses with $\frac{2}{3}$ of his earnings, he had \$12.36 remaining; what were his daily wages?
Ans. 80 cents.

SUPPLEMENTARY WRITTEN EXERCISES.

To be omitted unless otherwise directed.

42. Mr. Bowman gave $12\frac{1}{4}$ bushels of potatoes, at 30 cents a bushel, for butter worth $18\frac{3}{4}$ cents a pound; how much butter did he get? *Ans.* $20\frac{1}{2}$ pounds.

43. James Allen bought land for $10,260, and sold it so as to gain $\frac{1}{19}$ of the cost, the gain being $3 an acre; how many acres did he buy? *Ans.* 180 acres.

44. Mr. Robinson bought 100 yards of double width sheeting, he sold $\frac{1}{4}$ of it to Mr. Brown and $\frac{2}{3}$ of the remainder to Mr. Jones; what is the value of the remainder at $25\frac{1}{2}$ cents a yard? *Ans.* $11.58\frac{3}{4}$.

45. If at a certain time of day a pole 63 feet long casts a shadow $36\frac{3}{4}$ feet long, what is the length of a pole which casts a shadow $33\frac{1}{3}$ feet long? *Ans.* $57\frac{1}{3}$ feet.

46. A cistern has a capacity of $289\frac{1}{11}$ gallons, and has a pipe discharging into it $25\frac{1}{3}$ gallons per hour, and there is a leak through which it loses $5\frac{1}{7}$ gallons per hour; how long will it take to fill the cistern? *Ans.* $14\frac{7}{22}$ hours.

47. Mr. Williams, laying in a stock of goods, invests $\frac{1}{3}$ of his money in flour, $\frac{1}{4}$ in sugar and molasses, $\frac{1}{12}$ in tea and coffee, and the remainder, $540, in sundry other groceries; what was the whole amount invested? *Ans.* $1620.

48. What is the value of $\dfrac{\frac{4}{7}\text{ of }\frac{3}{5}}{5\frac{1}{14}-4\frac{3}{56}}-\dfrac{\frac{5}{11}\text{ of }1\frac{13}{15}}{7\frac{1}{12}-6\frac{1}{2}}$? *Ans.* 1.

49. What is the value of $\dfrac{\dfrac{1+\frac{1}{3}}{1-\frac{1}{3}}}{\dfrac{1\times\frac{1}{3}}{1\div\frac{1}{3}}}$? *Ans.* $37\frac{1}{2}$.

50. Value of $\dfrac{6\frac{1}{4}+5\frac{1}{2}}{6\frac{1}{4}-5\frac{1}{2}}+\dfrac{6\frac{1}{4}\times 5\frac{1}{2}}{6\frac{1}{4}\div 5\frac{1}{2}}-\dfrac{7\frac{1}{2}}{8}$? *Ans.* $43\frac{5}{12}$.

51. Value of $\left(\dfrac{\frac{1}{2}+\frac{1}{10}}{6\frac{1}{3}}+\dfrac{5\frac{5}{7}}{4\frac{4}{5}}\right)\div 11\times\dfrac{8\frac{1}{12}}{15\frac{2}{3}}$? *Ans.* $\frac{4}{15}$.

52. Value of $\dfrac{1-\frac{1}{3}}{2}\times\dfrac{2-\frac{1}{3}}{3}\times\dfrac{3-\frac{1}{3}}{4}+\dfrac{4-\frac{1}{3}}{5}$? *Ans.* $\frac{11}{15}$.

INTRODUCTION TO DECIMALS.

MENTAL EXERCISES.

1. If a unit is divided into 10 equal parts, what is one of these parts called?

2. If one-tenth is divided into 10 equal parts, what is one part called? 2 parts? 3 parts?

3. If $\frac{1}{100}$ is divided into 10 equal parts, what is one part called? 3 parts? 4 parts?

4. What part of 4 is 4 tenths? of 5 is 5 tenths? of 5 tenths is 5 hundredths? of 6 tenths is 6 hundredths?

5. What is $\frac{1}{10}$ of 3 hundredths? What part of 3 hundredths is 3 thousandths? of 6 thousandths is 6 ten-thousandths?

6. In the number 4444, the 4 units is what part of the 4 tens? the 4 tens is what part of the 4 hundreds?

7. A term in units place denotes what part of the value which it does in tens place?

8. A term in tens place denotes what part of the value which it does in hundreds place?

9. By the same law, a term to the right of units place would denote what part of its value in units place? *Ans.* One-tenth.

10. How may we indicate that a term is at the right of units place? *Ans.* By placing a dot (.) at the right of units place.

11. How will you express $2\frac{5}{10}$ in this manner? *Ans.* 2.5.

12. What does the dot between the 2 and the 5 denote? *Ans.* That 2 is in units place and 5 in tenths place.

13. In the expression 11.1, the 1 at the right of units denotes what part of a unit?

14. In the expression 11.11, the second term at the right of the period denotes what part of a tenth? what part of a unit?

15. How may we then write tenths, hundredths, etc., without a denominator? *Ans.* By writing them at the right of units.

16. What shall we call the first place at the right of units? Second place? Third? Fourth? Fifth?

17. Write without a denominator 2 tenths; 3 tenths; 4 tenths; 6 tenths; 8 tenths; 3 hundredths; 5 hundredths; 1 thousandth; 3 thousandths; 6 thousandths.

18. Read the following expressions: 4.6; 25.45; 26.34; 18.05; 25.235; 36.205; 46.008.

19. These fractions arising from the successive division by 10 are called *decimal fractions*. The term decimal is derived from *decem*, meaning *ten*.

SECTION V.

DECIMAL FRACTIONS.

195. A **Decimal Fraction** is a number of *tenths, hundredths, thousandths*, etc.

196. A **Decimal Fraction** is usually expressed by placing a point before the numerator and omitting the denominator; thus .5 expresses $\frac{5}{10}$; .05 expresses $\frac{5}{100}$; .005 $\frac{5}{1000}$, etc.

197. The **Symbol** of a decimal is the *period*, called the *decimal point*, or *separatrix*. It indicates the decimal, and *separates* decimals and integers.

198. The places at the right of the decimal point are called *decimal places*. The first place to the right of the decimal point is *tenths*, the second place is *hundredths*, etc. Thus, .245 expresses 2 *tenths*, 4 *hundredths*, and 5 *thousandths*.

199. The method of expressing decimal fractions arises from the method of notation for integers, and is a continuation of it. This beautiful law, as applied to integers and fractions, is exhibited in the following

NOTATION AND NUMERATION TABLE.

Hund.-millions.	Ten-millions.	Millions.	Hund.-thousands.	Ten-thousands.	Thousands,	Hundreds.	Tens.	Units.	*Decimal Point.*	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hund.-thousandths.	Millionths.	Ten-millionths.	Hund.-millionths.	Billionths
6	6	6	6	6	6	6	6	6	.	6	6	6	6	6	6	6	6	6
9th.	8th	7th.	6th.	5th.	4th.	3d.	2d.	1st.		1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.	9th.
Integers.										Decimals								

200. A **Decimal** is a decimal fraction expressed by the method of decimal notation; as, .5, .25, etc.

201. **A Pure Decimal** is one which consists of decimal figures only; as, .345.

202. **A Mixed Decimal** is one which consists of an integer and a decimal; as 4.35.

203. **A Complex Decimal** is one which contains a common fraction at the right of the decimal; as, .45$\frac{1}{2}$.

NOTES.—1. The first treatise upon decimals was written by Stevinus, and published in 1585.

2. The decimal point, Dr. Peacock thinks, was introduced by Napier, the inventor of logarithms, in 1617, though De Morgan says that Richard Witt made as near an approach to it as Napier.

EXERCISES IN NUMERATION.

1. Read the decimal .45.

SOLUTION.—This expresses 4 tenths and 5 hundredths, or since 4 tenths equals 40 hundredths, and 40 hundredths plus 5 hundredths equal 45 hundredths, it may also be read 45 hundredths. Hence the following rules:

Rule I.—*Begin at tenths, and read the terms in order towards the right, giving each term its proper denomination.*

Rule II.—*Read the decimal as a whole number, and give it the denomination of the last term at the right.*

NOTE.—In the second method we may determine the denominator by numerating *from* the decimal point, and the numerator by numerating *towards* the decimal point.

Read the following decimals:

2. .35	7. .507	12. 5.0708
3. .76	8. .605	13. 8.75006
4. .84	9. .7605	14. 43.30027
5. .031	10. .0576	15. 756.00279
6. .162	11. .00312	16. 372.000086

EXERCISES IN NOTATION.

1. Express 25 thousandths in the form of a decimal.

SOLUTION.—25 thousandths equal 20 thousandths plus 5 thousandths, or 2 hundredths and 5 thousandths; hence we write the 5 in the third or thousandths place, 2 in the second or hundredths place, and fill the vacant tenths place with a cipher, and we have .025. Hence the following rules:

Rule I.—*Place the decimal point, and then write each term so that it may express its proper denomination, using ciphers when necessary.*

Rule II.— *Write the numerator, and then place the decimal point so that the right-hand term shall express the denomination of the decimal, using ciphers when necessary.*

NOTE.—To avoid ambiguity, where integers and decimals occur in the same written number, a comma should be placed between them; thus, three hundred and seven ten-thousandths should be written .0307, but three hundred, and seven ten-thousandths, 300.0007.

Express the following in decimal form:

2. Twenty-five hundredths.
3. 2 tenths and 8 hundredths.
4. 7 tenths and 9 hundredths.
5. Twenty-five thousandths.
6. 4 tenths and 7 thousandths.
7. Seven tenths and 8 thousandths.
8. Five hundred, and 25 thousandths.
9. Three tenths and 7 ten-thousandths.
10. Seven hundredths and 9 ten-thousandths.
11. Three hundred, and 78 ten-thousandths.
12. Five tenths, 6 hundredths, and 7 hundred thousandths.
13. Four hundredths, seven ten-thousandths, and 6 hundred-thousandths.
14. Nine hundred and sixty-nine hundred-thousandths.
15. Two tenths and three millionths.
16. Five hundredths, six thousandths, and eight millionths.
17. Thirty-five thousand, and eight millionths.
18. Ninety-three hundred and seven ten-millionths.
19. Eighteen thousand and one hundred-millionths.
20. Two million, and 6 thousand and 9 hundred-millionths.

Express the following as decimals:

21. $\frac{7}{10}$. *Ans.* .7.
22. $\frac{93}{100}$. *Ans.* .93.
23. $\frac{76}{1000}$. *Ans* .076.
24. $\frac{302}{10000}$. *Ans.* .0302.
25. $124\frac{7}{100}$. *Ans.* 124.07.
26. $967\frac{104}{10000}$. *Ans.* 967.104.
27. $96\frac{202}{10000}$. *Ans.* 96.0209.
28. $75\frac{8017}{10000000}$. *Ans.* 75.008017.

PRINCIPLES.

1. *Moving the decimal point one place to the right, multiplies the decimal by* 10; *two places, multiplies by* 100; *etc.*

For, if the point be moved one place to the right, each figure will express ten times as much as before, hence the whole decimal will be ten times as great; etc.

2. *Moving the decimal point one place to the left, divides the decimal by* 10; *two places, divides by* 100; *etc.*

For, if the point be moved one place to the left, each figure will express 1 tenth of its previous value, hence the whole decimal will be only 1 tenth as great; etc.

3. *Placing a cipher between the decimal point and the decimal, divides the decimal by* 10.

For, this moves each figure one place to the right in the scale, in which case they express 1 tenth as much as before, and hence the decimal is only 1 tenth as great.

4. *Annexing ciphers to the right of a decimal does not change its value.*

For, each figure retains the same place as before, and hence expresses the same value as before, and consequently the value of the decimal is unchanged.

MENTAL EXERCISES.

1. How many tenths in 1? in $\frac{1}{2}$? in $\frac{1}{5}$? in $\frac{2}{5}$?
2. How many hundredths in $\frac{1}{2}$? in $\frac{2}{5}$? in $\frac{4}{5}$? in $\frac{9}{10}$? in $\frac{11}{50}$?
3. How many hundredths in $\frac{1}{4}$? in $\frac{3}{4}$? in $\frac{1}{25}$? in $\frac{4}{25}$? in $\frac{7}{25}$?
4. How many halves in .5? in .50? 5ths in .2? in .4?
5. How many 4ths in .25? in .75? eighths in .125? in .375?
6. Express as a common fraction and reduce to lowest terms, .5; .4; .8; 25; .50; .75; .80; .125; .625.

REDUCTION OF DECIMALS.

204. There are two cases of the reduction of decimals:

1st. To reduce decimals to common fractions.

2d. To reduce common fractions to decimals.

CASE I.

205. *To reduce a decimal to a common fraction.*

1. Reduce .45 to a common fraction

SOLUTION.—.45 expressed in the form of a common fraction is $\frac{45}{100}$, which reduced to its lowest terms equals $\frac{9}{20}$. Hence the following

OPERATION.

$.45=\frac{45}{100}=\frac{9}{20}$, *Ans.*

Rule.—*Write the denominator under the decimal, omitting the decimal point, and reduce the common fraction to its lowest terms.*

WRITTEN EXERCISES.

Reduce the following decimals to common fractions:

2.	.35.	*Ans.* $\frac{7}{20}$.	7.	9.75.	*Ans.* $9\frac{3}{4}$.
3.	.48.	*Ans.* $\frac{12}{25}$.	8.	12.725.	*Ans.* $12\frac{29}{40}$.
4.	.125.	*Ans.* $\frac{1}{8}$.	9.	16.075.	*Ans.* $16\frac{3}{40}$.
5.	.625.	*Ans.* $\frac{5}{8}$.	10.	5.064.	*Ans.* $5\frac{8}{125}$.
6.	3.25.	*Ans.* $3\frac{1}{4}$.	11.	17.0125.	*Ans.* $17\frac{1}{80}$.

12. Reduce the complex decimal $.2\frac{2}{3}$ to a common fraction.

SOLUTION.—$2\frac{2}{3}$ is $2\frac{2}{3}$ tenths, which, by writing the denominator, becomes $\frac{2\frac{2}{3}}{10}$, which equals $\frac{\frac{8}{3}}{10}$, or $\frac{8}{30}$, which reduced to its lowest terms equals $\frac{4}{15}$. Therefore, etc.

OPERATION.

$$.2\frac{2}{3} = \frac{2\frac{2}{3}}{10} = \frac{\frac{8}{3}}{10} = \frac{8}{30} = \frac{4}{15}$$ *Ans.*

Reduce the following to common fractions or mixed numbers:

13. $.8\frac{1}{3}$.	*Ans.* $\frac{5}{6}$.	18. $4.8\frac{1}{3}$.	*Ans.* $4\frac{5}{6}$.
14. $.16\frac{2}{3}$.	*Ans.* $\frac{1}{6}$.	19. $12.18\frac{3}{4}$.	*Ans.* $12\frac{3}{16}$.
15. $.25\frac{3}{5}$.	*Ans.* $\frac{32}{125}$.	20. $22.03\frac{1}{3}$.	*Ans.* $22\frac{1}{30}$.
16. $.45\frac{5}{6}$.	*Ans.* $\frac{11}{24}$.	21. $50.08\frac{5}{6}$.	*Ans.* $50\frac{53}{600}$.
17. \$0.16$\frac{2}{3}$.	*Ans.* \$$\frac{1}{6}$.	22. \$6.66$\frac{2}{3}$.	*Ans.* \$6$\frac{2}{3}$.

CASE II.

206. ***To reduce a common fraction to a decimal.***

1. Reduce $\frac{5}{8}$ to a decimal.

SOLUTION.—$\frac{5}{8}$ equals $\frac{1}{8}$ of 5. 5 equals 50 tenths; $\frac{1}{8}$ of 50 tenths is 6 tenths and 2 tenths remaining; 2 tenths equal 20 hundredths; $\frac{1}{8}$ of 20 hundredths equals 2 hundredths and 4 hundredths remaining; 4 hundredths equal 40 thousandths; $\frac{1}{8}$ of 40 thousandths is 5 thousandths. Therefore, $\frac{5}{8}$ equals .625.

OPERATION.

$\frac{5}{8} = \frac{1}{8}$ of 5

$= 8)5.000$

$.625$

Rule.—I. *Annex ciphers to the numerator, and divide by the denominator.*

II. *Point off as many decimal places in the quotient as there are ciphers used.*

NOTES.—1. In many cases the division will not terminate; the common fraction cannot then be exactly expressed by a decimal. Such decimals are called *interminate* or *infinite* decimals.

2. The symbol + annexed to a decimal indicates that it contains other decimal terms. The symbol — annexed to a decimal indicates that the last decimal term is increased by 1. This is often done when the next term is greater than 5.

WRITTEN EXERCISES.

Reduce the following common fractions to decimals:—

2. $\frac{1}{4}$.	*Ans.* .25.	8. $\frac{11}{15}$.	*Ans.* .733+.
3. $\frac{3}{4}$.	*Ans.* .75.	9. $\frac{15}{16}$.	*Ans.* .9375.
4. $\frac{3}{8}$.	*Ans.* .375.	10. $\frac{16}{25}$.	*Ans.* .64.
5. $\frac{7}{8}$.	*Ans.* .875.	11. $\frac{16}{17}$.	*Ans.* .9412—.
6. $\frac{3}{16}$.	*Ans.* .1875.	12. $\frac{18}{19}$.	*Ans.* .947368+.
7. $\frac{5}{16}$.	*Ans.* .3125.	13. $\frac{113}{256}$.	*Ans.* .44140625.

ADDITION OF DECIMALS.

207. **Addition of Decimals** is the process of finding the sum of two or more decimals.

MENTAL EXERCISES.

Add

1. $\frac{3}{10}$ and $\frac{5}{10}$.
2. 3 and .5.
3. .04 and .03.
4. .6 and .04.
5. 25 and .5.
6. .07 and .08.
7. .05 and .136.
8. 2.5 and 4.25.

WRITTEN EXERCISES.

1. What is the sum of 11.96, 25.075, 84.306, 90.728?

SOLUTION.—We write the numbers so that terms of the same order shall stand in the same column, and begin at the right to add. 8 thousandths, plus 6 thousandths, plus 5 thousandths, are 19 thousandths, which equals 1 hundredth and 9 thousandths; we write the 9 thousandths, and add the 1 hundredth to the next column: 2 hundredths, plus 7 hundredths, plus 6 hundredths, equal 15 hundredths, and the 1 hundredth added is 16 hundredths, which equals 1 tenth and 6 hundredths; we write the 6 hundredths. etc.

OPERATION.

11.96
25.075
84.306
90.728
212.069

Rule.—I. *Write the numbers so that terms of the same order stand in the same column.*

II. *Add as in whole numbers, and place the decimal point between the units and tenths of the sum.*

2. Find the sum of 79.76, 85.08, 36.125, 140.309.

Ans. 341.274.

3. Find the sum of 87.09, 58.37, 95.42, 237.675.

Ans. 478.555.

4. Add 18.79, 147.072, 856.709, 185.8761, 397.05784.

Ans. 1605.50494.

5. Add 59.874, 435.095, 672.328, 976.309, 8467.500843.

Ans. 10611.106843.

6. What is the sum of \$25$\frac{3}{4}$, \$37$\frac{1}{5}$, \$28.37$\frac{1}{2}$, \$50.06$\frac{1}{4}$, \$15 37$\frac{1}{2}$, \$57$\frac{3}{8}$, \$15$\frac{7}{8}$, and \$23.87$\frac{1}{2}$? *Ans.* \$253.88$\frac{3}{4}$.

7. Add 9 and 7 tenths, 41 and 8 hundredths, 75 and 54 hundredths, 128 and 187 thousandths. *Ans.* 254.507.

8. Add 187 and 5 thousandths, 49 and 9 hundred-thousandths, 1876 and 245 millionths, 187 ten-thousandths, and 999 ten-millionths. *Ans.* 2112.0241349.

9. Add 798 and 9 ten-thousandths, 17 millionths, 18 thousandths and 98 ten-millionths, 67 hundred-thousandths and 95 ten-millionths. *Ans.* 798.0196063.

10. Find the sum of 3 tenths, $6\frac{1}{2}$ hundredths, $4\frac{1}{4}$ thousandths, $3\frac{1}{5}$ ten-thousandths, and $6\frac{1}{8}$ hundred-thousandths.

Ans. .3696245.

SUBTRACTION OF DECIMALS.

208. Subtraction of Decimals is the process of finding the difference between two decimals.

MENTAL EXERCISES.

1. Subtract $\frac{4}{10}$ from $\frac{9}{10}$.
2. Subtract .4 from .9.
3. Subtract .06 from 08.
4. Subtract .05 from .55.
5. Subtract $\frac{5}{10}$ from .9.
6. Subtract .05 from .5.
7. Subtract .25 from .63.
8. Subtract .75 from 1.08.

WRITTEN EXERCISES.

1. From 972.163 take 856.235.

SOLUTION.—We write the numbers so that terms of the same order stand in the same column, and begin at the right to subtract. We cannot subtract 5 thousandths from 3 thousandths, hence we add ten thousandths to 3 thousandths, which equals 13 thousandths; 5 thousandths from 13 thousandths leaves 8 thousandths, which we write in the order of thousandths: since we have added 10 thousandths or 1 hundredth to the minuend, we must add 1 hundredth to the subtrahend; 1 hundredth and 3 hundredths are 4 hundredths; 4 hundredths from 6 hundredths leaves 2 hundredths, etc.

OPERATION.

$$\begin{array}{r} 972.163 \\ 856.235 \\ \hline 115.928 \end{array}$$

Rule.—I. *Write the subtrahend under the minuend, so that terms of the same order stand in the same column.*

II. *Subtract as in whole numbers, and place the decimal point between the units and tenths of the remainder.*

2. From 707.325 take 623.452. *Ans.* 83.873.

3. From 826.438 take 734.936. *Ans.* 91.502.

4. From 78.3057 take 29.084. *Ans.* 49.2217.

5. From 1230.207 take 384.1231. *Ans.* 846.0839.

6. From 2.07 take 1.432765. *Ans.* .637235.

7. From .3 take 3 hundred-millionths. *Ans.* .29999997.

8. From 1 and .001 take .01 and .000001. *Ans.* .990999.

9. From $2\frac{1}{2}$ take $2\frac{1}{2}$ thousandths and $2\frac{1}{2}$ billionths.

Ans. 2.4974999975

MULTIPLICATION OF DECIMALS.

209. Multiplication of Decimals is the process of finding the product when one or both factors are decimals.

MENTAL EXERCISES.

What is the value of

1. 2 times $\frac{1}{10}$? 8×.4?
2. 2 times .6? 5×.6?
3. $\frac{1}{10}$ times $\frac{7}{10}$? .5×.7?
4. $\frac{1}{10}$ times $\frac{4}{10}$? .4×.8?
5. $\frac{1}{10}$×.6? .6×.7?
6. 5×$\frac{1}{100}$? 5×.08?
7. $\frac{1}{10}$×$\frac{12}{100}$? .8×.12?
8. .6×.15? .12×.12?

9. How many places in the product of *units* multiplied by *tenths*? *Tenths* by *tenths*? *Units* by *hundredths*? *Tenths* by *hundredths*? *Hundredths* by *hundredths*?

10. How many decimal places in the product if there are two in the multiplicand and one in the multiplier? If there are two in each factor? If three in multiplicand and two in multiplier?

WRITTEN EXERCISES.

1. Multiply 7.23 by .46.

SOLUTION.—7.23 multiplied by 46 equals 332.58, and multiplied by 46 *hundredths* the product is 1 hundredth as great, which, by removing the decimal point two places to the left, becomes 3.3258. Hence, 7.23 multiplied by .46 equals 3.3258.

OPERATION.

7.23
.46
4338
2892
3.3258

SOLUTION 2D.—$7.23\times.46=\frac{723}{100}\times\frac{46}{100}=\frac{33258}{10000}=\frac{1}{10000}\times33258=3.3258$. From either of these solutions we derive the following

Rule.—*Multiply as in whole numbers, and from the right of the product point off as many decimal places as there are in both factors, prefixing ciphers when necessary.*

2. Multiply 27.13 by .67. *Ans.* 18.1771.
3. Multiply 43.08 by 2.36. *Ans.* 101.6688.
4. Multiply 79.52 by .019. *Ans.* 1.51088.
5. Multiply 8.534 by 20.074. *Ans.* 171.311516.
6. Multiply 123.107 by 1.52. *Ans.* 187.12264.
7. Multiply 512.073 by 35.08. *Ans.* 17963.52084.
8. Multiply 54.0079 by 7.072. *Ans.* 381.9438688.
9. Multiply 1.08096 by 3.5702. *Ans.* 3.859243392.
10. Multiply .03507 by .005873. *Ans.* .00020596611.
11. Multiply 2.0709 by .000246. *Ans.* .0005094414.
12. Value of $9\frac{1}{2}\times.08\frac{3}{4}\times10$? *Ans.* 8.3125.

13. Value of .5 of $\frac{7}{11} \times .05\frac{1}{2}$? *Ans.* .0175.

14. Value of .07 of $\frac{5}{8} \times 300 \times .09\frac{3}{7}$? *Ans.* 1.65.

15. Value of $\$25.6\frac{7}{8} \times .05\frac{1}{3} + \$15 \times 6.03\frac{1}{4}$?

Ans. $91.8575.

16. $\overline{200 + .02 - \frac{1}{3}} \times .8\frac{1}{4} + (.8\frac{1}{4} - .12\frac{7}{10} \times \frac{3}{4}$ of $7\frac{1}{5})$?

Ans. 164.7432.

17. Multiply 1 hundredth by 1 thousandth, and add 1 tenth to the product. *Ans.* .10001.

18. What is the product of one-tenth by one-tenth? one hundred by one-hundredth? one million by one-millionth?

DIVISION OF DECIMALS.

210. Division of Decimals is the process of finding the quotient when one or both terms are decimals.

MENTAL EXERCISES.

What is the value of

1. $\frac{4}{10} \div 2$? .4 ÷ 2?	**6.** $6 \div \frac{3}{10}$? 6 ÷ .3? 6 ÷ .03?
2. $\frac{24}{100} \div 6$? .24 ÷ 6?	**7.** .8 ÷ .4? 8 ÷ .4? 8 ÷ .04?
3. .50 ÷ 2? .5 ÷ 2?	**8.** 7.2 ÷ .6? .72 ÷ .6? .72 ÷ .06?
4. .150 ÷ 2? .15 ÷ 2?	**9.** 8.4 ÷ .7? .84 ÷ .7? .84 ÷ .07?
5. 5 ÷ 4? .5 ÷ 4? .05 ÷ 4?	**10.** 96 ÷ .8? 9.6 ÷ .8? .96 ÷ .8?

11. What is the product of *tenths* by *tenths*? What then is the quotient of *hundredths* by *tenths*? What is the product of *tenths* by *hundredths*? What is the quotient of *thousandths* divided by *tenths*?

12. If there are three decimal places in the multiplicand and two in the multiplier, how many in the product? If there are five in the dividend and three in the divisor, how many in the quotient?

WRITTEN EXERCISES.

1. Divide 12.3084 by 2.34.

SOLUTION.—Dividing as in whole numbers, we obtain the quotient 526; now, since the dividend is the product of the divisor and quotient, the number of decimal places in the dividend should equal the number in the divisor, plus the number in the quotient; hence the number of decimal places in the quotient equals the number of decimal places in the dividend, minus the number of places in the divisor; hence there should be *four* minus *two*, or *two* decimal places in the quotient, therefore the quotient is 5.26.

OPERATION.

2.34)12.3084(5.26
1170
608
468
1404
1404

SOLUTION 2d.—$12.3084 \div 2.34 = \frac{123084}{10000} \div \frac{234}{100} = \frac{123084}{10000} \times \frac{100}{234} = \frac{123084}{100 \times 234} = \frac{1}{100} \times \frac{123084}{234} = \frac{1}{100}$ of 526 = 5.26. From either of these solutions we derive the following

Rule.—I. *Annex ciphers to the dividend, if necessary to make the number of decimals equal to the number of decimal places in the divisor.*

II. *Divide as in whole numbers, annexing ciphers to the dividend when needed to continue the division.*

III. *Point off as many decimals in the quotient as the number of decimal places in the dividend exceeds the number in the divisor.*

NOTES.—1. When there are ciphers at the right of the divisor, cut them off, divide by the significant part, and then point off as many decimal places as before, plus the number of ciphers cut off.

2. Make complex decimals pure or divide them like common mixed numbers, or multiply both by the L. C. M. of the denominators, and then divide.

2. Divide 272.636 by 6.37. *Ans.* 42.8.
3. Divide 281.8585 by 3.85. *Ans.* 73.21.
4. Divide 57.85728 by 8.32. *Ans.* 6.954.
5. Divide 50.38218 by 67.8. *Ans.* .7431.
6. Divide 31.421154 by 85.43. *Ans.* .3678.
7. Divide 268.32615 by 4.705. *Ans.* 57.03.
8. Divide 2.3697792 by 34.56. *Ans.* .06857.
9. Divide 3411.3424 by 5.678. *Ans.* 600.8.
10. Divide 195.388698 by 32.185. *Ans.* 6.0708.
11. Divide 1678.543 by 185.79321. *Ans.* 9.03446+.

What is the value of

12. .065÷.026? *Ans.* 2.5.
13. .375÷.025? *Ans.* 15.
14. .008÷.04? *Ans.* .2.
15. .012÷.06? *Ans.* .2.
16. .004÷2.5? *Ans.* .0016.
17. 1÷.125. *Ans.* 8.
18. .51÷.015. *Ans.* 34.
19. 155÷.0625. *Ans.* 2480.
20. .00625÷25. *Ans.* .00025.
21. 25÷.00625. *Ans.* 4000.

22. $(2.04 \div 17 + \overline{47 \div 200} \times 5000) - \frac{7}{8}$? *Ans.* 1174.245.
23. $(789 - .789) \div (\overline{.75 - .075} \times .75 \text{ of } 8)$? *Ans.* 194.62.
24. $\overline{45 \times .181} + \overline{3.6 \times .3\frac{1}{3}} \div (3.5 - \frac{3.42}{7})$? *Ans.* $8.5436\frac{214}{401}$.
25. $(\$347.84 \div \$10.87) \times \overline{.0025 + .01\frac{3}{4}} \times 50$? *Ans.* 32.
26. $(\$1080 \times 3.27) \div \$10.90 - (\$790 \div \$3.95)$? *Ans.* 124.

WRITTEN PROBLEMS.

MISCELLANEOUS EXAMPLES.

1. What common fraction equals .00096? *Ans.* $\frac{3}{3125}$.

2. Add $\frac{.2}{2.7}$, $11\frac{\frac{1}{2}}{5.4}$, $2\frac{7}{45}$, and .1. *Ans.* $13\frac{19}{45}$.

3. Add $\frac{.4}{5}$, $\frac{2\frac{5}{8}}{5\frac{3}{4}}$, $\frac{2\frac{1}{4}}{45}$, and $\frac{2\frac{4}{9}}{1\frac{7}{18}}$. *Ans.* $2\frac{311}{400}$.

4. Multiply .0075 cwt. by 4.008. *Ans.* .03006 cwt.

5. Divide $13\frac{2}{3}\frac{2}{5}$ tons by $28\frac{\frac{1}{5}}{.7}$. *Ans.* $\frac{58}{110}$ tons.

6. Multiply 1.25 of $.8\frac{3}{4}$ of $9\frac{6}{25}$ by .8 of $\frac{\frac{7}{8}}{.2}$ of 8.75.
Ans. $307\frac{58}{256}$.

7. Divide $118\frac{1}{4}$ by $.04\frac{1}{2}$; also $2.4001\frac{3}{5}$ by $1.56\frac{1.25}{6\frac{1}{4}}$.
Ans. $2627\frac{7}{9}$; $1.53\frac{518}{781}$.

8. Divide $14\frac{\frac{2}{3}}{27}$ square yards by $11\frac{1\frac{2}{4}}{13\frac{1}{2}}$. *Ans.* $1\frac{469}{1808}$.

9. Divide £$240\frac{3}{4}$ by $\frac{3}{4}$ of $\frac{24\frac{1}{2}}{28}$ of $\frac{5}{7}$ of $1\frac{4}{5}$. *Ans.* £$285\frac{1}{3}$.

10. What number multiplied by $7\frac{3}{5}$ will give $6\frac{2}{3}$ for a product? *Ans.* $\frac{50}{57}$.

11. Divide seven millionths by twelve and a half ten-millionths. *Ans.* 5.6.

12. Add $\frac{1}{4}$, $\frac{8\frac{1}{2}}{36}$, .375, and .5, and multiply the sum by $6.2\frac{1}{2}$.
Ans. $8\frac{75}{144}$.

13. Subtract $\frac{3}{7}$ of 12 from $\frac{5}{8}$ of $\frac{8\frac{3}{4}}{\frac{2}{5}}$, and divide the remainder by $\frac{5}{7}$. *Ans.* $11\frac{391}{820}$.

14. Multiply the sum of .5 and $\frac{3\frac{1}{2}}{14}$ by the difference between $\frac{8\frac{1}{2}}{17}$ and $\frac{10}{26\frac{2}{3}}$. *Ans.* $\frac{3}{52}$.

15. Multiply $\frac{62\frac{1}{2}}{1000}$ by 25 millionths, and divide the product by 125 hundred-thousandths. *Ans.* .00125.

16. The product of two numbers is $\frac{4}{5}$, and one of them is $\frac{3}{4}$ of $\frac{4}{5}$ of 2; what is the other? *Ans.* $1\frac{1}{15}$.

17. Add 2.5 cwt., 1.25 cwt., 5 cwt., 4.375 cwt., 1.875 cwt., 2.5 cwt., 1.225 cwt., and 1.275 cwt. *Ans.* 20 cwt.

18. What number multiplied by $\frac{4}{5}$ of $\frac{1}{17}$ of $5\frac{1}{2}$ is equal to $\frac{3}{4}$ of $\frac{8}{11}$ of $27\frac{1}{2}$? *Ans.* $8\frac{7}{15}$.

19. Divide the sum of six thousandths and six millionths by their difference, to 6 decimal places. *Ans.* 1.002002 +.

20. From the sum of $\frac{2}{11}$, $\frac{3}{4}$, and $\frac{1}{3}$, take the remainder obtained by subtracting $\frac{7}{15}$ from $\frac{4}{13}$ of $\frac{5}{8}$ of $\frac{1}{2}$. *Ans.* $1\frac{5}{6}\frac{3}{6}\frac{5}{6}$.

21. What number must be divided by $\frac{3}{5}$ of $2\frac{2}{3}$ of $\frac{5\frac{1}{2}}{1\frac{2}{3}}$, to make a quotient equal to the value of $6\frac{2}{3}$ of $\frac{1\frac{1}{2}}{10}$ of 6? *Ans.* 24.

22. John had \$225$\frac{3}{8}$ and earned $\frac{1}{5}$ of $3\frac{1}{2}$ times \$88$\frac{1}{2}$ more; having lost part of his money, he found that he had \$186$\frac{3}{8}$ remaining; what amount did he lose? *Ans.* \$97$\frac{3}{8}$.

23. A ton of iron ore from the mines of Cornwall yields .65 of a ton of pure iron; how much iron will 578.8 tons of ore yield? *Ans.* 376.22 tons.

24. An oil refiner has on hand 16,745 gallons of coal oil; how many casks, each containing 42.5 gallons, can be filled with it? *Ans.* 394 casks.

25. How many dress patterns of 12.25 yards each, can be cut from a piece of French percale, containing 50 yards? *Ans.* 4+ patterns.

26. An engine pumped 41.25 barrels of 31.5 gallons each from a tank containing 1500 gallons; how many gallons remained? *Ans.* 200.625 gal.

27. I bought 3 loads of wood, the first containing 1.02 cords, the second 1.05 cords, the third .945 cords; what did it cost at \$3.60 a cord? *Ans.* \$10.854.

28. A dealer bought 1086 bushels of wheat; how many bins, each holding 20.25 bushels, will it fill, and how many bushel-bags can be filled from what remains?

Ans. 53 bins; 12.75 bags.

29. Two speculators bought 4320 acres of Western land,

which they divide so that one has .37½ and the other .62½ of it; how many acres had each? *Ans.* 1620; 2700.

30. How many lengths of 6-inch stove-pipe can be made from 87.48 pounds of Norway iron, if one length requires 3.24 pounds? *Ans.* 27.

31. Mr. Newlin paid $4000 for a truck farm, giving $76.25 an acre for 27.25 acres, and $85.75 for the remainder; how many acres did he buy? *Ans.* 49.66+ acres.

32. A man bought 345.75 tons of hay, at $16.25 a ton, ⅓ of which he sold at $17.75 a ton, and the rest at cost; how much was the gain? *Ans.* $172.87½.

33. Mr. Hartman bought a lot of wheat and sold .15 of it to one man, and .25 of it to another, and kept 572.85 bushels; how much did he buy? *Ans.* 954.75 bu.

34. A man devotes .12 of his income to charity, .25 for educating his children, .45 for household expenses, and saves the remainder, which is $284.76; required his income.

Ans. $1582.

35. How many cords of wood, at $5.12½ a cord, must I give for 91.25 bushels of wheat, at $1.40 a bushel, and 85 bushels of rye, at $1.25 a bushel? *Ans.* 45.66 cords.

36. The circumference of the fore wheel of a wagon is 12.75 feet, and of the hind wheel 14.25 feet; how much oftener does one turn than the other in going 5280 feet, or one mile? *Ans.* 43.59+ times.

37. A ship whose cargo was worth $25,000, being disabled in a storm, .45½ of the whole cargo was thrown overboard; what would a merchant lose who owned .25 of the cargo? *Ans.* $2843.75.

38. A grain dealer expended $6210 for grain, ⅓ of it being for wheat, at $1.25 a bushel, ¼ of it for corn, at $0.75 a bushel, and the remainder for rye, at $1.12½ a bushel; how many bushels of each kind did he purchase?

Ans. Wheat, 1656 bu.; Corn, 2070 bu.; Rye, 2300 bu.

UNITED STATES MONEY.

211. United States Money, or the currency of the United States, is expressed in the decimal system.

212. The several **denominations** and their relation to each other are presented in the following table:

TABLE.

10 mills equal 1 cent.	10 dimes equal 1 dollar.
10 cents " 1 dime.	10 dollars " 1 eagle.

NOTE.—¼ of a dollar = 25 cents; ½ of a dollar = 50 cents; ¾ of a dollar = 75 cents; ½ of a cent = 5 mills.

213. The **dollar** is the unit and is indicated by the symbol $; the eagle and dollar are read as a number of dollars. Thus $245 is read 245 dollars, instead of 24 eagles, 5 dollars.

214. The **dime** is one tenth of a dollar, and is expressed as tenths, the decimal point being placed between dimes and dollars. Thus $2.3 expresses 2 dollars and 3 dimes.

215. The **cent** is one tenth of a dime or one hundredth of a dollar, and is written in hundredths place. Thus $6.75 indicates 6 dollars 7 dimes and 5 cents. Dimes and cents, however, are usually read as a number of cents. Thus $6.75 is read 6 dollars and 75 cents.

216. Since dimes and cents are regarded as a number of cents, when the number of cents is less than 10, a cipher must be written in tenths place. Thus 3 dollars and 4 cents are written $3.04.

217. The **mill** is one tenth of a cent or one thousandth of a dollar, and is written in thousandths place. Thus $8.375 is read 8 dollars 37 cents and 5 mills.

NOTES.—1. In checks, notes, drafts, etc., cents are usually written as hundredths of a dollar in the form of a common fraction, as 12\frac{75}{100}$.

2 When the final result of a business computation contains mills, if 5 or more they are reckoned 1 cent, and if less than 5 they are rejected. Thus $7.187 would be reckoned as $7.19 and $3.162 as $3.16.

3. We used dollars and cents in treating the fundamental rules; we now give a more formal treatment of decimal currency, involving problems that pupils were not then prepared to solve.

EXERCISES IN NUMERATION.

Read the following:

1. $15.65.	5. $28.05.	9. $100.001.
2. $14.753.	6. $10.50.	10. $202.202.
3. $28.284.	7. $105.105.	11. $370.037.
4. $132.125.	8. $125.005.	12. $1030.001.

EXERCISES IN NOTATION.

Write on the slate or board

1. Seven dollars and twenty-five cents.
2. Twelve dollars, thirty cents, and five mills.
3. Twenty-five dollars, fifty-four cents, and five mills.
4. Thirty-four dollars, seven cents, and seven mills.
5. Nine eagles, six dollars, six cents, and eight mills.
6. Two hundred dollars, seven and one-half cents.
7. Forty-nine dollars, six dimes, and $7\frac{1}{2}$ cents.
8. Five hundred and thirty-eight dollars, $62\frac{1}{2}$ cents.

REDUCTION OF UNITED STATES MONEY.

218. **Reduction** is the process of changing a number from one denomination to another without altering its value.

219. From the explanation given we have the following

PRINCIPLES.

1. *To reduce cents to mills, annex one cipher.*
2. *To reduce dollars to cents, annex two ciphers.*
3. *To reduce dollars to mills, annex three ciphers.*
4. *To reduce cents to dollars, place the point two places from the right.*
5. *To reduce mills to dollars, place the point three places from the right.*

NOTE.—In reducing a number of dollars and cents to cents, etc., the separatrix should be removed; thus, $5.25 = 525 cents, and $8.755 = 8755 mills.

WRITTEN EXERCISES.

1. Reduce 5 dollars to cents.

SOLUTION.—In 1 dollar there are 100 cents, and in 5 dollars there are 5 times 100 cents, or 500 cents, or we annex two ciphers, as above directed.

2. Reduce 25 dollars to cents. *Ans.* 2500.
3. Reduce 24 cents to mills. *Ans.* 240.
4. Reduce 8 dollars to mills. *Ans.* 8000.
5. Reduce 20 dollars to mills. *Ans.* 20000.
6. Reduce 6 cents and 5 mills to mills. *Ans.* 65.
7. Reduce 12 dollars and 25 cents to cents. *Ans.* 1225.
8. Reduce 23 dollars and 5 cents to cents. *Ans.* 2305.
9. Reduce 1450 cents to dollars. *Ans.* $14.50.
10. Reduce 2700 cents to dollars. *Ans.* $27.
11. Reduce 5425 cents to dollars. *Ans.* $54.25.
12. Reduce 4170 mills to dollars. *Ans.* $4.17.
13. Reduce 250 mills to cents. *Ans.* 25 cents.
14. Reduce 865 mills to cents. *Ans.* 86½ cents.
15. Reduce 13875 mills to dollars, cents, and mills. *Ans.* $13.875.
16. Reduce 185326 mills to dollars, cents, and mills. *Ans.* $185.326.
17. How many dollars, cents, and mills in 235 dollars, 2 cents, and 5 mills? *Ans.* $235.025.
18. How many dollars, cents, and mills in 150 dollars, 10 cents, and 5 mills? *Ans.* $150.105.

FUNDAMENTAL OPERATIONS.

220. Since **United States Money** is expressed in the decimal scale, all the operations may be performed as in decimals.

Rule.—*To add, subtract, multiply, or divide in United States money, proceed according to the corresponding operations in decimals.*

WRITTEN EXERCISES.

1. Find the sum of $62½ and 62½ cents. *Ans.* $63.12½.
2. From $25 take 25 cents and 5 mills. *Ans.* $24.74½.
3. Require the sum of $18¾, 18¾ dimes, 18½ cents, and 18 mills. *Ans.* $20.828.
4. From 1 dollar take 1 mill; from $505 take 505 cents. *Ans.* $0.999; $499.95.

5. Subtract $6\frac{1}{2}$ cents from $6\frac{1}{2}$ dollars, and add the remainder to $6\frac{1}{2}$ dimes. *Ans.* \$7.08$\frac{1}{2}$.

6. What must a stock-dealer pay for 27 young cows at \$18$\frac{3}{4}$ apiece? *Ans.* \$506.25.

7. A drover sold the government 33 horses at the rate of \$125$\frac{1}{4}$ apiece; what did he receive? *Ans.* \$4133.25.

8. If a cattle-feeder received \$250.25 for the sale of 13 cows, what did they bring him a head? *Ans.* \$19.25.

9. I paid \$36.25 for 5 barrels of flour; what would 17 barrels have cost me at the same rate? *Ans.* \$123.25.

10. A country merchant paid \$96 for a lot of muslins, at 16 cents a yard; how many yards did he buy? *Ans.* 600.

11. How many cords of wood can be bought for \$1553.25, at the rate of \$4.75 per cord? *Ans.* 327 cords.

12. A carpenter earned in 4 months \$240; how many days did he labor if his wages were \$2.50 a day? *Ans.* 96.

13. Divide \$16$\frac{3}{4}$ by 12 cents; divide \$48 by 6 dimes; divide 18$\frac{3}{4}$ dimes by 15 mills. *Ans.* $139\frac{7}{12}$; 80; 125.

14. A lady bought a watch for \$55$\frac{1}{2}$, a chain for \$22$\frac{3}{4}$, a key for \$6$\frac{1}{4}$, and sold them all at a gain of \$6$\frac{3}{4}$; what did she receive for them? *Ans.* \$91.25.

15. A teacher bought a book for \$2$\frac{1}{2}$, an inkstand for 62$\frac{1}{2}$ cents, some paper for \$1$\frac{1}{4}$, a map for \$4$\frac{3}{4}$, a globe for \$5$\frac{1}{2}$, and handed the clerk a ten-dollar-bill and a five-dollar bill; how much change should he receive? *Ans.* \$0.37$\frac{1}{2}$.

16. Mr. Benton bought 12 hogsheads of molasses of 63 gallons each, at the rate of 42$\frac{1}{2}$ cents a gallon, and sold it at 50 cents a gallon; what was the gain? *Ans.* \$56.70.

17. A lady bought 3 yards of muslin at 6$\frac{1}{4}$ cents a yard, 7 yards of linen at 87 cents a yard, and handed out a \$10 bill; what was her change? *Ans.* \$3.72.

18. There was sold one day, in New York, Low Grade extra flour, amounting to \$15,000 at \$6.25 per barrel; how many barrels were sold? *Ans.* 2400 barrels.

19. A lady in furnishing her house bought 3 sets of chairs at \$7.25 a set, 2 tables at \$5.25 apiece, 3 rocking-chairs at

$4.65 apiece, and 45 yards of carpet at $1.75 a yard; what was the amount of the bill? *Ans.* $124.95.

20. A dealer bought 8 barrels of turpentine, each containing 31.5 gallons, at $1.12½ a gallon, and sold it for $1.37½ a gallon; what did he gain? *Ans.* $63.

21. A man bought a boat load of coal for $250, and by retailing it at $5.75 a ton, he gained $37.50; how many tons in the load? *Ans* 50 tons.

22. The charge for sending a telegram from New York to Harrisburg is $.40 for 10 words, and 5 cents for each additional word; what would a dispatch of 24 words cost me? *Ans* $1.10.

23. Thomas Williams & Co. sold 420 bushels of new ungraded corn at 60 cents per bushel, and received in exchange 120 bushels of oats at 43 cents per bushel, and No. 1 Minnesota wheat, at $1.67 per bushel; how many bushels did they receive? *Ans.* 120 bushels.

COMMERCIAL TRANSACTIONS.

221. In **Commercial Transactions** there are ordinarily three quantities considered, the *quantity*, the *price*, and the *cost*.

222. The **Quantity** is the amount bought or sold, estimated by the number of times it contains the *unit of measure*.

223. The **Price** is the value of one of the units of measure of any commodity. The *Cost* is the value of the whole quantity.

224. An **Aliquot Part** of a number is the whole or mixed number which will exactly divide that number.

ALIQUOT PARTS OF $1.

5 cents = $\$\frac{1}{20}$.	16⅔ cents = $\$\frac{1}{6}$.
6¼ cents = $\$\frac{1}{16}$.	20 cents = $\$\frac{1}{5}$.
8⅓ cents = $\$\frac{1}{12}$.	25 cents = $\$\frac{1}{4}$.
10 cents = $\$\frac{1}{10}$.	33⅓ cents = $\$\frac{1}{3}$.
12½ cents = $\$\frac{1}{8}$.	50 cents = $\$\frac{1}{2}$.

225. The simple operations of finding price, cost, and quantity have already been sufficiently indicated, and we shall here consider only a few special cases.

CASE I.

226. ***To find the cost of a quantity, the price being an aliquot part of $1.***

1. What cost 48 yards of muslin at $12\frac{1}{2}$ cents a yard?

SOLUTION.—At $1 a yard, the cost would be $48; hence at $12\frac{1}{2}$ cents, which is $\frac{1}{8}$ of $1, the cost will be $\frac{1}{8}$ of $48, or $6. Hence the

OPERATION.

$$8)\overline{48}$$
$$\overline{6}$$

Rule.—*Take such a fractional part of the given quantity as the price is of* $1.

2. What cost 12 pieces of calico, each containing 32 yards, at $6\frac{1}{4}$ cents a yard? *Ans.* $24.

3. What cost 4 bales of Sea Island cotton, each containing 300 lb., at $16\frac{2}{3}$ cents a pound? *Ans.* $200.

4. Mrs. Wilson bought a bag of Rio coffee, containing 40 lb., at $33\frac{1}{3}$ cents a lb., and 10 lb. of crushed sugar, at $12\frac{1}{2}$ cents a lb.; what was the cost of both? *Ans.* $14.58\frac{1}{3}$.

5. Bought 20 yards of black cashmere, at $1.37\frac{1}{2}$ a yard, 20 yards of paper muslin, at 10 cents a yard, and 3 dozen crocheted buttons, at 25 cents a dozen; what was my bill?

Ans. $30.25.

CASE II.

227. ***To find the cost, the quantity and the price of*** 100 ***or*** 1000 ***being given.***

1. What is the cost of 6501 feet of poplar boards, at $17.25 a thousand?

SOLUTION.—If 1000 feet cost $17.25, 1 foot will cost $\frac{1}{1000}$ of $17.25, and 6501 feet will cost 6501 times $\frac{1}{1000}$ of $17.25, which is the same as $\frac{1}{1000}$ of 6501 times $17.25, which by multiplying and cutting off three places in the product, we find is $112.14.

OPERATION.

$$\begin{array}{r} 17.25 \\ 6501 \\ \hline 112.14225 \end{array}$$

Ans. $112.14.

Rule.—*Multiply the price by the quantity, and point off in the product two places for price per hundred, or three places for price per thousand.*

NOTE.—The price per hundred or per thousand is expressed thus ℔ Ċ, or ℔ M.

2. What is the value of 12,500 shingles at $6.25 ℔ M?

Ans. $78.12½.

3. What is the cost of 7877 feet of hemlock scantling, at $3.37½ ℔ M? *Ans.* $26.58.

4. What is the cost of 17,225 bricks, at $7.75 ℔ M, and 1560 lb. of sheet lead, at $10.12½ ℔ C? *Ans.* $291.44

5. A grocer sold one day 9760 pounds of wheat flour, at $4.25 ℔ C; what was the amount of the sale?

Ans. $414.80.

6. Messrs. Wood & Co. burn in their store in a year 62,560 cubic feet of gas, at $4 50 ℔ M; what is their gas bill for a year? *Ans.* $281.52.

7. If a compositor is paid 45 cents per thousand ems, what will he receive for setting up a book of 560 pages of 1120 ems each? *Ans.* $282.24.

8. Mr. Barr receives for making cigars $7.50 ℔ M; what will be his profit on 43,780 cigars, if he pays $5.75 for the amount of tobacco required to make a thousand?

Ans. $76.61½.

CASE III.

228. ***To find the price of*** **100** ***or*** **1000,** ***the quantity and cost being given.***

1. If I sell 4256 feet of boards for $62.776, what is the price ℔ M?

SOLUTION.—If 4256 feet cost $62.776, 1 foot will cost $\frac{1}{4256}$ of $62.776, and 1000 feet will cost 1000 times $\frac{1}{4256}$ of $62.776, which, dividing and removing the decimal point three places to the right, we find is $14¼. Hence the

OPERATION.
$62.776 ÷ 4256 = $14¼.

Rule.—*Divide the cost by the quantity, and remove the decimal point in the quotient two places to the right for price per hundred, or three places for price per thousand.*

2. I paid $7.59⅜ for 675 pickets for the fence around my garden; what was the price ℔ C? *Ans.* $1.12½.

3. Mr. Shirk sold 10,540 cedar rails for $553.35; what was the price ℔ C? *Ans.* $5¼.

4. If the cost of stereotyping a book of 320 pages of 850 ems each is $340, what is the cost per 1000 ems?

Ans. $1.25.

5. I retail envelopes at 12 cents a pack, gaining 3 cents on each pack of 24; what did they cost me ℔ M?

Ans. $3.75.

CASE IV.

229. ***To find the cost, the quantity and the price of a ton of*** **2000** ***pounds being given.***

1. At $75.45 a ton, what will be the cost of 7247 lb. of railroad iron?

SOLUTION.—Dividing $75.45, the price of a ton, by 2, we have $37.72½, the price of 1000 lb.; and proceeding as in Case II., we have $273.39 as the price of 7247 lb.

OPERATION.

2)75.45

37.72½

7247

273.39307½

Rule.—*Multiply half the price of a ton by the quantity, and remove the decimal point three places to the left.*

NOTE.—To find the price of a ton, cost and quantity being given, *divide the cost by the quantity, multiply the quotient by 2, and remove the decimal point three places to the right.*

2. A farmer sold 7375 pounds of hay at $13½ a ton; what did he receive for it? *Ans.* $49.78.

3. A porcelain manufacturer bought 5620 pounds of clay at $17.25 a ton; what did it cost him? *Ans.* $48.47.

4. A peddler sold to a paper-mill 1728 pounds of rags at $48 per ton; what did he receive? *Ans.* $41.47.

5. If 7240 pounds of bone dust cost $93.215, what is the price per ton? *Ans.* $25.75.

6. A coal dealer received $20.88 for 5760 pounds of Lehigh red ash coal; what was the price per ton? *Ans.* $7.25.

7. Shipped from Pittsburgh, on the Panhandle R. R., 84,761 pounds of pig iron @ $3.75 per ton, and 97,486 pounds of steel rails at $2.50 per ton; what was the charge?

Ans. $187.03.

8. Messrs. Wilson & Co. bought 36,850 pounds of lime at $3.60 a ton, and sold it at 22½ cents per hundred; what was the profit on the transaction? *Ans.* $16.58.

BILLS AND ACCOUNTS.

230. A **Bill** is a written statement of goods sold, services rendered, etc., giving the place, date, names of the parties, and the price, quantity, and cost of each item.

231. An **Invoice** is a full statement, sent to a purchaser or agent, at the time the goods are forwarded, containing the marks, contents, and prices of each package, the charges paid, and the mode of forwarding. The terms *Invoice* and *Bill* are often used interchangeably.

232. The **Footing** of a bill is the amount of its items. To *extend* an item is to write its cost in the proper column. A bill is *receipted* when the person to whom it is due, or his agent, writes at the bottom of the bill "Received Payment," and signs his name.

233. An **Account Current** is a written statement of the business transactions between two parties for a given time. The party who owes is the *Debtor;* the party owed is the *Creditor.*

234. To **Balance an Account** we find the difference of the footings of the two sides and add it to the smaller side, so that the two amounts are equal.

NOTES.—1. The abbreviation *Dr.* signifies *debit* or *debtor;* *Cr.*, *credit*, or *creditor;* @ signifies *at*, and denotes the price of the *unit of measure;* % stands for *account;* ¢ for *cents;* No. or № for *number;* pcs. for *pieces*, and 62^2, 18^3, 6^1, signify respectively $62\frac{2}{3}$, $18\frac{3}{4}$, $6\frac{1}{4}$; 3 pcs. sheeting 42, 42^1, 45^3 signifies that the pieces contain respectively 42, $42\frac{1}{4}$ and $45\frac{3}{4}$ yards.

2. *Accounts current* are frequently made out every month, and are then called *monthly statements*, and generally contain only the amounts bought at each date, bills of the several items having been furnished at the time of buying.

3. Deductions are often made in bills, sometimes from the retail price of the items, and sometimes from the amount of the bill. Deductions from the retail price are generally made to customers buying in considerable quantities, and deductions from the amount are made for cash payments or payments within a short specified time, the prices in such bills being mostly wholesale. The symbol %, meaning *hundredth*, is frequently used; thus, less 6% means 6 *hundredths* deducted.

Rule I.—*In making out a* BILL, *extend the several items and take their sum.*

Rule II.—*In an* ACCOUNT, *find the difference between the debit and credit amounts.*

235. Required the footings and balances of the following bills and accounts:

BALTIMORE, *July* 15, 1876.

JOHN B. WYLIE & Co.,

Bought of MACNEIL, WALCOTT & CO.

24	Prs.	Men's French Calf Boots, @	$10	
65	"	Women's Enameled "	1.87½	
75	"	" Fox'd Gaiters,	1.50	
88	"	Boys' Kip Boots,	2.50	
110	"	" Brogans,	1.25	
90	"	Misses' Rubber Shoes,	.75	
60	"	Women's Arctics,	2.25	
		Less ⅓		827 50

Rec'd Payment,
MACNEIL, WALCOTT & Co.

WILMINGTON, DEL., *April* 27, 1876.

MR. JAMES LANE,
To A. B. HAYES, DR.

For Setting Boiler and Excavating,		100	00
" 13½ Days Work of Bricklayer,	@$4		
" 5½ " " " Laborer,	1.12½		
" 1 Load Sand,		2	50
" 15 Bushels Lime,	.38		
" 500 Dark Stretchers,	@$14 ⅞ M		
" 500 Salmon Bricks,	" 8 " "		
		179	39

PROVIDENCE, R. I., *June* 1, 1876.

MR. JAMES HEDGE,
Bought of GLADDING & BRO.,

1876						
Apr	3	1	Webster Unabridged,	1/10	12	
	7	4	Leisure Day Rhymes,	¼	2	00
May	2	3	Eight Cousins,	½	1	50
	6	2	Ticknor's Life and Letters,	½	3	50
	"	4	Mysterious Island,	½	3	00
	29	3	Farm Legends,	1/10	2	50
	"	2	Three Brides,	½	1	75
						38 61

Rec'd payment,
JOHN B. TIBBETS,
for GLADDING & BRO.

PHILADELPHIA, *Oct.* 1, 1876.

MRS. GEORGE GORDON,

Bought of WALKER & BROUGHER.

Terms: 30 *days.*

2	Pr.	W. G. Ewers and Basins				1.40				
2		" Cov'd Soaps				50				
2		" Brush Trays				50				
½	Doz.	" Bowls				1.75				
1	"	" Mugs						1	50	
2	"	Stem Wines				1.00				
2	"	Glass Peppers				1.25				
2	"	Individual Salts				50				
3	"	Oval Glass Dishes	1-3 in 100	1-6 in 150	1-7 in 200					
		Crate and Porterage						1	25	
									18	43

NOTE.— $\frac{\text{1-3 in}}{100}\ \frac{\text{1-6 in}}{150}\ \frac{\text{1-7 in}}{200}$ means that there are 1 dozen 3 inch dishes @ $1, 1 dozen 6 inch @ $1.50, and 1 dozen 7 inch @ $2.

STATEMENT OF ACCOUNT.

NEW YORK, *April* 7, 1876.

MR. HENRY CADY,

In ℅ with BADEAU & LOCKWOOD,

1875						
		DR.				
Oct.	9	To 15 Drums of Figs,	2 25			
"	30	" 9 Boxes Oranges,	4 75			
Dec.	3	" 12 " Lemons,	5 25			
1876						
Jan.	29	" 4 Barrels Dried Apples,	6 75			
Feb.	"	" 10 " " Peaches,	5 25			
Mar.	18	" 30 lb. Leghorn Citron,	25			
		CR.				
1876						
Jan.	11	By 2 bbl. Superfine State Flour,	6 00			
"	"	" 200 lb. Buckwheat "	.03½			
Feb.	1	" 1 Firkin Butter, 56 lb.,	28			
"	29	" 100 lb. Corn Meal, ₱ C.,			1	35
Mar.	1	" Cash,			108	00
		Balance due Badeau & Lockwood,				82 47

J. T. WAY & Co.

In % with WOOD, MARSH & HEYWARD,

DR.							CR.		
1875				1875					
Sept.	25	To 1 pc. Blue Anchor Cloth 38¹ yd. @ $5.62²,		Oct.	2	By 1 pc. Irish Linen 17¹ yd.@57²¢,			
"	30	" 2 pcs. Black Cloth 42 yd.@$7.65,		"	20	" 3 pcs. Plaid Jaconets 60 yd.@37²¢			
Oct.	8	" 1 pc. Doeskin Cassimere 24 yd. @ $1.87²,		Nov.	30	" 1 pc. Swiss Mull 20 yd.@32²¢,			
Nov.	9	" 2 pcs. Black Rib'd Cassimere 63 yd. @$1.63,		Jan.	2	" Note for Balance @30 da.,		739	82
Dec.	14	" 2 pcs. Fisher's Tweed 64¹ yd. @75¢,							
"	30	" 2 pcs. Brown Corduroy 41¹ yd.@ $1.12²,							

WRITTEN EXERCISES.

236. Let pupils make out bills, accounts, and invoices in proper form, from the following statements:

1. The Lafayette Restaurant ordered Oct. 10, 1876, from Thos. Bradley, the following: 67 lb. Ribs of Beef @ 20¢; 2 Kidneys, @ 15¢; 3 Lambs, @ $5; 3 Calves' Heads, @ 75¢; 8 Sweetbreads, @ 30¢; 2 Calves' Livers, @ 70¢; 37 lb. Veal, @ 17¢; make out the bill. *Ans.* $41.04.

2. Eugene G. Blackford delivered the following, June 12, 1876, at the Continental Hotel: 15 Salmon, @ 30¢; 37 Lobsters, @ 10¢; 5 Fresh Mackerel, @ 12¢; 7 Spanish Mackerel, @ 20¢; 2 Shad, @ 40¢; 12 Soles @ 10¢; 9 Sea-bass, @ 10¢; 15 Sheepheads, @ 12¢; 3 doz. Soft Crabs, @ 50 ¢; 1½ doz. Frogs, $1.75; extend the items and find the footing of the bill. *Ans.* $19.03.

3. James Thomson bought of William Wilson, Pittsburgh, Jan. 20, 1873, 1 lb. Citric Acid, $1.55; 5 lb. Chloride Lime. @ 8¢; jar 20¢; 10 lb. Epsom Salts, @ 5¢; 3 lb. Gum Shellac, @ 60¢; 1 doz. Wilson's Cod Liver Oil, @ $7; ½ doz. Mishler's Herb Bitters, @ $8; 2 lb. Carbolic Acid Crystals, @ $1.75; what was the amount of the bill? *Ans.* $18.95.

SUPPLEMENTARY PROBLEMS.

To be omitted unless otherwise directed.

CINCINNATI, O., *Sept. 25th*, 1874.

MR. JOHN WALLER,

Bought of TIERNEY & GARDEN.

Terms: 30 *days.*

½	Dozen Gents Blk. Fur Hats,	16 50	
½	" " " " "	19 50	
1	" " " " "		18 00
½	" Boys " Sax. "	9 00	
½	" " " " "	12 00	
½	" Children's Fancy "	6 00	
½	" Ladies Trimmed "	15 00	
½	" " Untrimmed "	7 50	
	Less 6%,		57 11

Received payment,

TIERNEY & GARDEN,

Per J. W. M.

PITTSBURGH, *Nov.* 1, 1876.

Messrs. WILEY & MORTON,

Bought of J. G. HERR & CO.

2	Doz. Rubber Long Combs,	4 50	
¼	Gro. Horn " $\frac{3.50}{\text{No. }8}$, $\frac{4.50}{10}$, $\frac{6.00}{12}$		
1½	Doz. Tooth Brushes,	1 75	
1½	" Jet Sets,	7 25	
1	Gro. Rubber Coat and Vest Buttons ea. 1.00 & 75,		1 75
½	Gt. " Black Suspender "	1 62	
40	Yd. Jaconet,	20	
14	" Irish Linen,	67	
3	Pcs. Silk Velvet Ribbon $\frac{65}{1\frac{1}{4}}$, $\frac{72\frac{1}{2}}{2\frac{1}{2}}$, $\frac{1.35}{4}$, $\frac{2.00}{6}$,		
	Less $\frac{1}{10}$,		47 07

Rec'd payment,

J. G. HERR & Co.

NOTE.—This bill shows another method of abbreviating entries of similar items. ¼ Gro. Horn Combs $\frac{3.50}{\text{No. }8}$, $\frac{4.50}{10}$, $\frac{6.00}{12}$, means that there are ¼ gross No. 8 @ $3.50, ¼ gross No. 10 @ $4.50, and ¼ gross No. 12 @ $6. In the last item 1¼, 2½, etc., are the width of the ribbon, the upper numbers being the price as before.

SUPPLEMENTARY PROBLEMS.

To be omitted unless otherwise directed.

236. Let pupils make out bills, accounts, and invoices from the following statements:

4. Tyndale & Co., Philadelphia, sold to Mrs. John Smith, Dec. 31, 1874, the following articles: 1 Soup Tureen, @ $3.50; 2 Sauce Tureens, @ $1.25; 1 doz. Tulip Goblets, @ $1.40; 1 doz. Individual Salts, @ 50¢; 2 Glass Pitchers, @ 62½¢; 3 Oval Glass dishes, @ 50¢; 1 Glass Nappy, @ 75¢; 4 Cov'd Dishes, @ $1.25; 3 doz. Stoneware Plates,—1 doz. 6 in., @ $1.25, 1 doz. 7 in., @ $1.40, 1 doz. 8 in., @ $1.60; required the bill, receipted. *Ans.* $20.65.

5. Mrs. Amelia Watson, Newark, N. J., presented the following bill to James Haven, March 1, 1875; Board for 4 weeks, @ $9; fuel and light 4 weeks, @ $1.50; washing 5½ doz. @ $1. Mr. Haven presented the following bill to Mrs. Watson at the same date: February 5th, 15 lb. Tea, @ 75¢; 10 lb. Coffee, @ 35¢; February 9th, 25 lb. Granulated Sugar, @ 12½¢; 5 lb. Brown Sugar, @ 10¢; 1 barrel No. 1 Mackerel, $25; February 26th, 5 lb. Butter, @ 50¢, and 1½ dozen Eggs, @ 42¢. Make out both bills, receipting the smaller and crediting the amount upon the other. *Ans.* Bal. $1—.

6. August 25, 1875, Franklin S. Fuller, of Memphis, Tenn., purchased of Kuhn & Furst, Philadelphia, 25 boxes Raisins, @ $\frac{10}{\$3.25}$; $\frac{15}{\$2.55}$; 350 lb. Currants, @ $\frac{100}{7\frac{1}{2}¢}$, $\frac{250}{7¢}$; 150 lb. Dates, @ 5½¢; 100 lb. Turkish Prunes, @ 8½¢; 125 lb. French Prunes, @ $\frac{50}{15¢}$, $\frac{75}{12\frac{1}{2}¢}$; 12 bunches of Baracoa Bananas, @ $1.75; and gave a due bill for amount. Make out bill and receipt it. *Ans.* $150.

7. Mr. Thomas Walker, of Aiken, S. C., bought of Hess, Rogers and Chambers, Philadelphia, the following articles, Oct. 14, 1876: 3 doz. Ladies' Berlin Gloves, @ $2.25; 1 doz. Ladies' White Silk do., @ $5.50, 1 doz. do. Berlin ½ Gauntlets, @ $3.75; 1 doz. Colored Buck do., @ $15; 1 doz. Ladies' Black Jouvin Kid Gloves, @ $16; ½ doz. White do., @ $15; 2 doz. Gents' Buck Driving Gauntlets, @ $16.50; 2 doz. Gents' White Kid Gloves, @ $11; 1 doz. Child's White Cotton Hose $\frac{80}{3}$, $\frac{90}{3^2}$, $\frac{95}{4}$, $\frac{100}{4^2}$, $\frac{110}{5}$; make out bill for amount, deducting 20%.

Ans. $91.40.

SECTION VI.

DENOMINATE NUMBERS.

237. A **Denominate Number** is a concrete number in which the unit is a *measure;* as 3 *feet*, 4 *pounds*, etc.

238. A **Measure** is a unit by which quantity of *magnitude* or *continuous* quantity is estimated numerically; as, a *yard*, a *pound*, etc.

239. A **Compound Denominate Number** is a number which expresses several different units of the same kind of quantity; as, 4 yd., 2 ft., and 11 in.

240. The **Terms** of a compound number are the *numbers of its different units.* Thus the *terms* in the example given are 4 yd., 2 ft., and 11 in.

241. **Similar Compound Numbers** are compound numbers which express the same kind of quantity.

242. **Denominate Numbers** may be embraced under eight distinct classes, as follows:

1. Value.
2. Weight.
3. Length.
4. Surface.
5. Volume.
6. Capacity.
7. Time.
8. Angles.

NOTE.—Concrete numbers are of two classes: 1st, those in which the unit is *natural*; 2d, those in which it is *artificial.* Natural units are such as exist in nature, and artificial units are those which are agreed upon to measure quantity of magnitude. The latter are called *denominate numbers.*

MEASURES OF VALUE.

243. The **Value** of anything is its worth, or that property which makes it useful or estimable.

244. **Money** is the measure of the value of things. It is of two kinds, coin and paper money.

245. **Coin**, or **Specie**, is metal prepared and authorized by government to be used as money.

246. Paper Money consists of printed promises to pay the bearer a certain amount, duly authorized to be used as money.

247. Currency is whatever circulates as money. It is of two kinds, *specie currency* and *paper currency*.

248. Legal Tender is a term applied to money which is required by law to be accepted in payment of debts.

249. An **Alloy** is a baser metal compounded with either gold or silver for the purpose of rendering it harder and more durable. In coinage, the alloy is considered as having no value.

UNITED STATES MONEY.

250. United States, or **Federal Money**, is the legal currency of the United States.

TABLE.

10 mills (m)	= 1 cent	.	ct.
10 cents	= 1 dime	.	d.
10 dimes	= 1 dollar	.	$.
10 dollars	= 1 eagle	.	E.

I. NAME.—United States money is so called because it is the money of the United States. It is called *Federal Money* because it was the money of the Federal Union. It was adopted by Act of Congress, Aug. 8, 1786.

II. TERMS.—The term *dollar* is supposed to be from *Dale* or *Daleburg*, a town where it was first coined; *dime* is from the French *disme*, meaning *a tenth*; *cent* is from the Latin *centum*, *a hundred*; *mill* is from the Latin *mille*, *a thousand*; *eagle* is from the name of the national bird. The cent was proposed by Robert Morris, and named by Thomas Jefferson.

III. UNIT.—The *unit* is the *gold dollar*. The currency is founded upon the decimal system, dimes, cents, and mills being written as decimals. This gives great simplicity to the operations.

IV. COINS.—The coins are of *gold*, *silver*, *nickel*, and *bronze*. The *gold coins* are the *double eagle*, *eagle*, *half-eagle*, *quarter-eagle*, *three dollars*, and *one dollar*. The *silver coins* are the *trade dollar*, *half-dollar*, *quarter-dollar*, *twenty-cent piece*, and *dime*. The *nickel coins* are the *three-cent* and *five-cent pieces*. The *bronze coin* is the *cent*. The silver half-dime and three-cent piece, the bronze two-cent piece, the nickel cent, and the old copper cent and half-cent, although still seen in circulation, are no longer coined. The mill has never been a coin; it is merely a convenient name for the tenth part of a cent.

V. COMPOSITION.—The gold and silver coins consist of 9 parts of pure metal and 1 part alloy. The alloy of the silver coin consists of pure copper; the alloy of the gold coin consists of silver and copper, the silver not to exceed $\frac{1}{10}$ of the alloy. The nickel coins contain $\frac{1}{4}$ nickel and $\frac{3}{4}$ copper. The bronze coins consist of 95 parts copper, and 5 parts tin and zinc.

NOTE.—The *Notes* on the tables are not to be memorized or recited, unless otherwise directed.

VI. Gold coins are a legal tender for any amount; silver coins, of the present coinage, for any amount not exceeding $5 in any one payment; bronze and nickel coins for any amount not exceeding 25 cents in any one payment.

MENTAL EXERCISES.

1. How many cents in $\frac{1}{2}$? $\frac{1}{10}$? $\frac{3}{4}$? $\frac{2}{5}$? $\frac{3}{5}$? $\frac{4}{5}$? $\frac{1}{8}$?

2. What part of a dollar is 10 cts.? 12½ cts.? 20 cts.? 25 cts.? 16⅔ cts.? 33⅓ cts.? 37½ cts.? 50 cts.? 62½ cts.? 75 cts.? 83⅓ cts.?

3. What part of 5 eagles is 15 dimes? what part of 12 cents is $\frac{1}{2}$ of a dime?

4. How many eagles in 60 dollars? in 400 dimes? in 8500 cents? in 25,000 mills?

ENGLISH, OR STERLING MONEY.

251. English, or **Sterling Money,** is the legal currency of England.

TABLE.

4 farthings (far. or qr.)	=1 penny .	d.
12 pence . .	=1 shilling .	s.
20 shillings . .	=1 pound or sovereign,	£
21 shillings . .	=1 guinea .	G.

I. NAME.—The term *Sterling* is supposed to be derived from *Easterling*, the name given to early German traders, who came from the east to England. Their money was called Easterling Money, which was contracted into *Sterling Money*.

II. TERMS.—The term *farthing* is a modification of "four things," the old English penny being marked with a cross so deeply impressed that it could be broken into two or four pieces, called respectively *half-penny* and *four things*. The *Pound*, as a measure of value, was derived from the pound as a measure of weight, 240 pence formerly weighing a pound. The *guinea* is so called because the gold of which it was first made came from Guinea, in Africa.

III. SYMBOLS.—The symbols £., s., d., qr., are the initials of the Latin words *libra*, *solidus*, *denarius*, and *quadrans;* signifying respectively, pound, shilling, penny, and quarter.

IV. UNIT.—The *unit* is the *pound*, represented by the sovereign and £1 bank note. Its value by late act of Congress is fixed at $4.8665.

V. COINS.—The coins are of three classes: *gold*, *silver*, and *copper*. The *gold coins* are the *sovereign* (=£1), and *half sovereign* (=10 s.), *guinea* (=21 s.) and *half guinea* (=10 s. 6 d.). The *silver coins* are the *crown* (=5 s.), the *half crown* (=2 s. 6 d.), the *florin* (=2 s.), the *shilling*, and the *six-penny*, *four-penny*, and *three-penny* pieces. The *copper coins* are the *penny*, *half-penny*, and *farthing*.

VI. COMPOSITION.—The standard for gold coins is 22 carats fine, that is, 11 parts pure gold, and 1 part alloy. The standard for silver is 37 parts pure silver and 3 parts alloy, hence the silver coins are $\frac{37}{40}$ pure, and $\frac{3}{40}$ copper. Pence and half-pence are made of pure copper.

MENTAL EXERCISES.

How many

1. Farthings in 6 pence?
2. Pence in 8 shillings?
3. Shillings in 108 pence?
4. Shillings in 25 pounds?
5. Pounds in 480 shillings?
6. Pence in 124 guineas?

7. What part of 2 pence is 6 farthings? What part of 3 shillings is 5 pence?

8. What part of 16 pence is $\frac{1}{3}$ of a shilling? What part of a guinea is $\frac{1}{7}$ of a pound?

WRITTEN EXERCISES.

1. How many pence in £16 11 s. 10 d.?

SOLUTION.—In one pound there are 20 shillings, and in £16 there are 16 times 20 shillings, which, increased by 11 s., are 331 shillings. In one shilling there are 12 pence, and in 331 shillings there are 331 times 12 pence, which, increased by 10 pence, equals 3982 pence.

OPERATION.

£	s.	d.
16	11	10
20		
331		
12		
3982d. *Ans.*		

2. How many pounds, shillings, and pence in 4367 pence?

SOLUTION.—There are 12 pence in one shilling, hence in 4367 pence there are as many shillings as 12 is contained times in 4367, which are 363 shillings and 11 d. remaining. There are 20 shillings in one pound, hence in 363 shillings there are as many pounds as 20 is contained times in 363, which are £18 and 3 shillings remaining. Hence in 4367 d. there are £18 3 s. 11 d.

OPERATION.

d.
12)4367
2|0)36|3 - 11 d.
£18 - 3 s.
Ans. £18 3 s. 11 d.

3. How many farthings in 9 s. 8 d. 3 far.? *Ans.* 467 far.

4. Farthings in £19 17 s. 11 d. 2 far.? *Ans.* 19102 far.

5. Shillings, pence, and farthings, in 7859 far.?

Ans. 163 s. 8 d. 3 far.

6. Pounds, shillings, etc., in 58763 far.?

Ans. £61 4 s. 2 d. 3 far.

CANADA MONEY.

252. The **Currency of Canada** is nominally the same as that of the United States, the table and denominations being the same.

253. The decimal currency was adopted in 1858, the Act taking effect in 1859, previous to which their currency was the same as the English.

I. COINS.—The coins consist of silver and copper. The *silver coins* are the 50-*cent piece*, the 25-*cent piece*, the *shilling* or 20-*cent piece*, the *dime*, the *half-dime*. The *copper coin* is the *cent*.

IV. VALUE.—The coins are nominally equal to the corresponding coins of United States money, but the intrinsic value is a little less. The *eagle* of the United States is the legal tender for sums of $10 and upwards.

III. COMPOSITION.—The silver coins consist of 925 parts silver and 75 parts copper; or 37 parts silver to 3 parts copper, the same as the English silver coins.

FRENCH MONEY.

254. French Money is the legal currency of France. The *unit* is the *franc*, whose value is 19.3 cents.

255. The **Franc** is divided into tenths and hundredths, called respectively *decimes* and *centimes*. The *decime*, like our dime, is not used in business calculations, but is expressed by *centimes;* thus, instead of 5 decimes we say 50 centimes.

GERMAN MONEY.

256. The **German Empire** has adopted a new and uniform system of coinage.

257. The **Unit** is the *mark* (*Reichsmark*) worth 23.85 cents, and this is divided into 100 *pfennige*.

WRITTEN EXERCISES.

1. How many dollars in 25.50 francs? *Ans.* $4.9215.
2. How many francs in $256? *Ans.* 1326.42+fr.
3. How many dollars in 753 marks 15 pfennige? *Ans.* $179.626+.
4. How many marks in $456.25? *Ans.* 1913—marks.
5. Bought an opera-glass for 20 francs; what did it cost in United States money? *Ans.* $3.86.

MEASURES OF WEIGHT.

258. Weight is the measure of the force by which bodies are naturally drawn towards the earth.

259. There are three kinds of weight in common use. *Troy Weight*, *Apothecaries' Weight*, and *Avoirdupois Weight*.

TROY WEIGHT.

260. Troy Weight is used in weighing gold, silver, jewels, liquors in philosophical experiments, etc.

TABLE.

24 grains (gr.)	= 1 pennyweight, . .	pwt.
20 pennyweights	= 1 ounce, ...	oz.
12 ounces	= 1 pound, ...	lb.

I. NAME.—The term *Troy* is said to be derived from *Troyes*, the name of a town in France, where this weight was first used in Europe. It was brought from Cairo in Egypt during the crusades of the 12th century.

II. TERMS.—The term *pound* is from the Latin *pendo*, to *bend* or *weigh*. The term *ounce* is from the Latin *uncia*, a *twelfth part*, the ounce being one-twelfth part of a pound. The *pennyweight* was the weight of the old English silver penny. The term *grain* originated in the custom of using a number of grains of wheat for the weight of a penny. These grains were taken from the middle of the ear, and well dried, thirty-two at first, and afterwards twenty-four, being used to make a pennyweight.

III. SYMBOLS.—The symbol *oz.* is thought to be from the Spanish word *onza*, signifying *ounce*; though Webster derives it from the use of the termination ʒ to express abbreviations, which was afterwards changed to *z*, *lb.* is from *libra*, the Latin for pound. *Pwt.* is a combination of *p.* for *penny*, and *wt.* for *weight*.

IV. UNIT.—The *standard unit* of weight is the *Troy pound*. It is equal the weight of 22.794377 cubic inches of distilled water, at the temperature of 39.83° Fahrenheit, barometer at 30 inches.

MENTAL EXERCISES.

How many

1. Grains in 6 pwt.? 5 pwt.?
2. Pwt. in 72 gr.? in 144 gr.?
3. Pwt. in 7 oz.? 9 oz.?
4. Ounces in 100 pwt.? 60 pwt.?
5. Ounces in 9 lb.? 11 lb.?
6. Pounds in 120 oz.? 144 oz.?
7. What part of a pound Troy are 4 oz.? 6 oz.? 8 oz.?
8. How many parts pure gold in a ring 18 carats fine?

WRITTEN EXERCISES.

How many

1. Grains in 7 pwt. 15 gr.? *Ans.* 183 gr.
2. Grains in 5 oz. 14 pwt. 13 gr.? *Ans.* 2749 gr.
3. Lb., oz., and pwt. in 758 pwt.? *Ans.* 3 lb. 1 oz. 18 pwt.
4. Oz., pwt., and gr. in 6275 gr.? *Ans.* 13 oz. 1 pwt. 11 gr.
5. Grains in 19 lb. 11 oz. 17 pwt. 19 gr.? *Ans.* 115147 gr.
6. Pounds, oz., etc., in 18759 gr.?

Ans. 3 lb. 3 oz. 1 pwt. 15 gr.

APOTHECARIES' WEIGHT.

261. Apothecaries' Weight is used in prescribing and mixing dry medicines. Medicines are bought and sold by Avoirdupois Weight.

TABLE.

20 grains (gr.)	= 1 scruple,	℈.
3 scruples	= 1 dram,	ʒ.
8 drams	= 1 ounce,	℥.
12 ounces	= 1 pound,	℔.

I. NAME.—The name arises from the weight being used by *apothecaries.*

II. TERMS.—The term *scruple* is from the Latin *scrupulus*, a *little stone.* The term *dram* is from the Greek *drachma*, *a piece of money.*

III. SYMBOLS.—The symbols have been supposed to be modifications of the figure 3, suggested by there being 3 scruples in a dram. Another supposition is that they are from inscriptions upon the ancient monuments of Egypt.

IV. UNIT.—The *Unit* is the *pound*, and is identical with the Troy pound, as are also the ounce and grain, the ounce being differently divided.

V. Physicians use the Roman notation in writing prescriptions, using the small letters, preceded by the symbols, and writing *j* for *i* when it terminates a number. Thus, 12 gr. is written gr.xij.; 2 scruples, ℈ij. ℞ is an abbreviation for *recipe*, *take*; $\bar{a}$ or $\bar{aa}$ (from the Greek ἀνά) means, *of each*; *ss.* for *semis* or *half*, as ℈ivss. means 4½ scruples; *P.* for *particula*, or *little part*; *P. aeq.* for *equal parts*; *q. p.*, *quantum placet*, *as much as you please.*

MENTAL EXERCISES.

How many

1. Grains in 7℈? 11℈?
2. Scruples in 9ʒ? 16ʒ?
3. Drams in 24℈? 96℈?
4. Drams in 5℥? 7℥?
5. Ounces in 88ʒ? 96ʒ?
6. Pounds in 108℥? 168℥?

7. How many doses of 8 grains each, can be made from an ounce of quinine?

WRITTEN EXERCISES.

How many

1. Drams in 7℔. 5 ℥? *Ans.* 712ʒ.
2. Pounds and ounces in 239ʒ? *Ans.* 19℔. 11℥.
3. Scruples in 19℔. 8ʒ 5ʒ 2℈? *Ans.* 5681℈.
4. Grains in 47℔. 10ʒ 7ʒ 2℈ 19gr.? *Ans.* 275999gr.
5. Pounds, etc., in 92375gr.? *Ans.* 16℔. 3ʒ. 1℈ 15gr.
6. How many pills of 4gr. each, can be made from 3ʒ 2℈ of calomel? *Ans.* 55.

AVOIRDUPOIS WEIGHT.

262. Avoirdupois Weight is used for weighing everything except jewels, precious metals, etc.

TABLE.

16 ounces	. .	= 1 pound,	. . lb.
100 pounds	. .	= 1 hundredweight,	cwt.
20 hundredweight	. .	= 1 ton,	. . T.

I. NAME.—The term *Avoirdupois* is probably from the French *avoir du poids, to have weight.*

II. TERMS.—The term *ton* is from the Saxon *tunne*, a *cask.* The origin of the other terms has already been given. The symbol *cwt.* is from *centum, hundred*, and *weight.* The term *dram* has been used for $\frac{1}{16}$ of an ounce, but is obsolete, fractions of an ounce being used in business transactions.

III. UNIT.—The *unit* is the *pound.* It is derived from the Troy pound, and contains 7000 grains Troy. It is equal to the weight of 27.7015 cubic inches of water at 39.83° Fah., the barometer being at 30 inches.

IV. In Great Britain 28 lb. equal 1 qr., 112 lb. equal 1 cwt., and 2240 lb. equal 1 ton. These are called the *long hundred* and *long ton;* they were formerly used in this country, but are now used only at the custom-houses in invoices of English goods, in the wholesale iron and plate trade, and in wholesaling and freighting coal from the coal mines of Pennsylvania.

V. The following denominations are frequently used:

25 lb.	of	powder	make 1 barrel.	100 lb.	of	raisins	make 1 cask.
56 "		butter	" 1 firkin.	196 "		flour	" 1 barrel.
84 "		"	" 1 tub.	200 "		pork, beef, or fish	1 barrel.
100 "		grain or flour	" 1 cental.	240 "		lime	" 1 cask.
100 "		dry fish	" 1 quintal.	280 "		salt at N.Y.S. works	1 barrel.
100 "		nails	" 1 keg.	600 "		rice	1 barrel.

MENTAL EXERCISES.

How many

1. Ounces in 8 lb.? 12 lb.?
2. Pounds in 112 oz.? 9 cwt.?
3. Cwt. in 4800 lb.? 1000 T.?
4. Tons in 2200 cwt.? 2500 cwt.?
5. How many cwt. in $\frac{1}{2}$ of a ton? in $\frac{1}{4}$ of a ton?
6. How many pounds in 50 cwt.? in 75 cwt.? in 80 cwt.?

WRITTEN EXERCISES.

How many

1. Pounds in 5 T. 16 cwt. 75 lb.? *Ans.* 11675 lb.
2. Tons, etc., in 7897 pounds? *Ans.* 3 T. 18 cwt. 97 lb.
3. Ounces in 29 cwt. 73 lb. 15 oz.? *Ans.* 47583 oz.
4. Ounces in 6 T. 83 lb. 13 oz.? *Ans.* 193341 oz.
5. Tons in 117309 ounces? *Ans.* 3 T. 13 cwt. 31 lb. 13 oz.
6. Which is heavier, and how much, a pound of lead or a pound of gold?

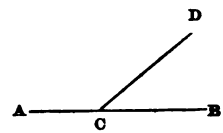
7. Which is heavier, and how much, an ounce of feathers or an ounce of silver?

MEASURES OF LENGTH.

263. Measures of Length are used in measuring length, breadth, height, distance, etc.

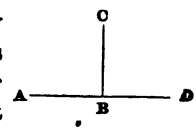
264. A Line is that which has length without breadth or thickness. It is estimated by ascertaining how many times it contains some definite length, regarded as a unit of measure.

265. An **Angle** is the opening between two lines which diverge from a common point. Thus ACD and DCB are angles.



266. The **Vertex** of an angle is the point from which the two lines diverge; thus, C is the vertex of the angle BCD.

267. A Right Angle is formed by one line perpendicular to another; as ABC or CBD. One line is *perpendicular* to another when it makes the two adjacent angles equal.



LONG MEASURE.

268. Long Measure is used for the general purposes of measuring length and distances.

TABLE.

12 inches (in.) . . .	=1 foot, ...	ft.
3 feet	=1 yard, ...	yd.
5½ yards, or 16½ feet .	=1 rod, ...	rd.
320 rods	=1 statute mile, .	mi.
3 miles	=1 league, . . .	lea.
69.16 miles (69⅛ nearly)	=1 degree of latitude, or of longitude at the equator.	deg. or °

1 TERMS.—The term *inch* is from *uncia*, a *twelfth*; *foot* is from the human foot; *yard* was a *rod* or *shoot*; *rod* is from a *measuring stick* or *rod*: *furlong*, which has now become obsolete, is from *fur*, *furrow*, and

lang, *long*, the *length of a furrow*; *mile* is from *mille passuum*, 1000 paces; *span* is the space measured from the end of the thumb to the end of the little finger extended; *cubit*, from the elbow to the end of the middle finger; *fathom*, the length of the two arms extended. The ancient yard of England is said to have been determined by the length of the arm of King Henry I.

II. UNIT.—The *standard unit* of length is the *yard*, from which all other measures of length, and also those of capacity, weight, etc., are derived. It is identical with the Imperial yard of Great Britain, which, under William IV., was declared to be fixed by dividing a pendulum, which vibrates seconds in a vacuum, at the level of the sea, at 62° Fah., in the latitude of London, into 391393 equal parts, and taking 360000 of these parts for the yard. Subsequent scientific experiments have proved that such a standard is impracticable.—*See Brooks's Philosophy of Arithmetic.*

III. THE MILE.—The *geographic* or *nautical mile* is equal to 1 minute of one of the great circles of the earth; hence it equals $\frac{1}{60}$ of $\frac{1}{360}$ of the circumference of the earth, which equals about 1.15 statute miles. The English mile is the same as that of the United States. 3 statute miles make a *land league*; 3 nautical miles make a *nautical league*.

IV. OTHER MEASURES.—The following denominations are frequently used: in clock-making, 6 *points* = 1 *line*, and 12 *lines* = 1 *inch*; in measuring the foot, 3 *barleycorns* or *sizes* = 1 inch; in measuring the height of horses, 4 inches = 1 *hand*, the measure being taken directly over the fore-shoulder; 1 *span* = 9 inches; 1 *common cubit* = 18 inches, and 1 *sacred cubit* = 21.888 inches; 1 *pace* = 3.3 feet; a *knot* is equal to a nautical mile. Formerly 40 rods made 1 *furlong* and 8 furlongs one mile, but these are seldom used.

MENTAL EXERCISES.

How many

1. Feet in 72 in.? 14 yd.?
2. Yards in 36 ft.? 10 rd.?
3. Rods in 22 yd.? 2 fur.?
4. Furlongs in 200 rd.? 7 mi.?
5. Miles in 320 fur.? 9 lea.?
6. Yards in 36 inches? 72 in.?

7. What part of 1 yard are 2 ft.; and what part of 2 furlongs are 5½ yards?

8. How high is a horse which measures 15¼ hands in height?

WRITTEN EXERCISES.

1. Reduce 27 ft. 8 in. to inches. *Ans.* 332 in.
2. 3 rods 4 yd. 2 ft. to feet. *Ans.* 63½ ft.
3. 567 inches to rods. *Ans.* 2 rd. 4 yd. 2 ft. 3 in.
4. 243 rd. 7 in. to inches. *Ans.* 48121 in.
5. 19870 ft. to miles. *Ans.* 3 mi. 244 rd. 1 yd. 1 ft.
6. 277 rd. 4 yd. to yards. *Ans.* 1527½ yd.
7. 70287 in. to miles. *Ans.* 1 mi. 34 rd. 5 yd. 1 ft. 3 in.
8. 3 mi. 265 rd. 7 in. to inches. *Ans.* 242557 in.
9. If the Atlantic is 3000 miles wide, how many feet is that, and how many steps, 3 feet long, would a person take to walk the distance? *Ans.* 5280000 steps.

SURVEYORS' LINEAR MEASURE.

269. Surveyors' Linear Measure is used by surveyors and engineers in measuring the dimensions of land, distances, etc.

TABLE.

7.92 inches (in.)	. .	=	1 link,	. .	li.
100 links	. . .	=	1 chain,	. .	ch.
80 chains	. . .	=	1 mile,	. .	mi.

I. NAME.—*Gunter's Chain* is named after the reputed inventor, Edmund Gunter, an English mathematician, born 1581.

II. UNIT.—The unit is a chain called *Gunter's Chain*, which consists of 100 links, and is 4 rods, 66 feet, or 792 inches long.

III. The denomination *rods* is seldom used by surveyors, distances being represented in chains and links. Since each link is $\frac{1}{100}$ of a chain, the number of links is generally expressed as a decimal; thus, 5 chains and 47 links are written 5.47 chains. Engineers generally use a chain 100 feet long.

MARINERS' AND CLOTH MEASURES.

270. Mariners' Measure is used by seamen in measuring distances, the depth of the sea, etc. *Cloth Measure* is used for measuring cloth, ribbons, etc.

MARINERS' MEASURE.	CLOTH MEASURE.		
6 feet =1 fathom.	1 yard	=	36 inches.
120 fathoms =1 cable length.	$\frac{1}{2}$ yard	=	18 inches.
880 fathoms =1 mile.	$\frac{1}{4}$ yard	=	9 inches.
	$\frac{1}{8}$ yard	=	$4\frac{1}{2}$ inches.

I. The *foot* and *yard* of these two measures are the linear foot and yard. The *nail* in Cloth Measure is obsolete. At the custom-houses the yard is divided into tenths, hundredths, etc.

II. In the old table of Cloth Measure there were given 3 qrs. = 1 Ell Flemish; 5 qrs. = 1 Ell English; 6 qrs. = 1 Ell French; 6 qrs. $1\frac{1}{2}$ in. = 1 Ell Scotch.

MENTAL EXERCISES.

How many

1. Inches in a chain?
2. Feet in a chain? a fathom?
3. Feet in 6 fathoms?
4. Feet in a cable length?

WRITTEN EXERCISES.

1. Reduce 4 miles to links. *Ans.* 32000 li.
2. Reduce 7 mi. 63 ch. to links. *Ans.* 62300 li.
3. Reduce 7856 fathoms to miles. *Ans.* 8 mi. 816 fath.
4. Reduce 98763 in. to chains. *Ans.* 124 ch. 70 li. $\frac{4}{9}$ in.

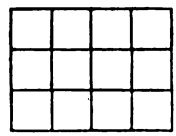
MEASURES OF SURFACE.

271. A **Surface** is that which has length and breadth without thickness.

272. A **Square** is a plane surface which has four equal sides and four right angles.

273. All **Surfaces** are *measured* by ascertaining the number of times they contain a small square regarded as the unit of measure.

Thus, in the surface in the margin there are three rows of squares, each row containing 4 squares; hence there are 3 times 4 or 12 squares in all; and since these make up the entire surface, the measure of the surface, called its *area*, is 12 square units.



SURFACE OR SQUARE MEASURE.

274. **Surface or Square Measure** is used in measuring surfaces, as land, boards, amount of painting, papering, plastering, paving, etc.

TABLE.

144 square inches (sq. in.)	=	1 square foot,	sq. ft.
9 square feet . . .	=	1 square yard,	sq. yd.
30¼ square yards, or 272¼ square feet,	=	1 perch or sq. rod,	P.
160 perches . . .	=	1 acre, . .	A.
640 acres . . .	=	1 square mile,	sq. mi.

I. TERMS.—*Perch* is from the French *perche*, a *pole*; *acre* was primarily an open plowed or sowed field.

II. UNIT.—The unit for land is the *acre*: for other surfaces it is usually the *square yard*.

III. The *Perch* is a surface equal to a *square rod*. The *rood* is found now only in old title-deeds and surveys; it is equal to 40 perches.

IV A square piece of land, measuring 209 feet, or about 70 paces on each side, equals very nearly one acre.

MENTAL EXERCISES.

How many

1. Square inches in 3 sq. ft.? 5 sq. ft.?
2. Square feet in 720 sq. in.? 7 sq. yd.?
3. Square yards in 144 sq. ft.? 5 P.?
4. Perches in 121 sq yd.? 12 R.?

5. Roods in 160 P.? 15 A.?

6. Acres in 84 R.? 3 sq. mi.

7. What is the difference between 3 feet square and 3 square feet? between 4 inches square and 4 square inches?

WRITTEN EXERCISES.

1. Reduce 9870 P. to acres. *Ans.* 61 A. 110 P.

2. 313 P. to square yards. *Ans.* 9468¼ sq. yd.

3. 3 A. 133 P. to square feet. *Ans.* 1668891/4 sq. ft.

4. 109 P. 17 sq. yd. to sq. in. *Ans.* 4295268 sq. in.

5. 238975 sq. ft. to acres.

Ans. 5 A. 77 P. 211 sq. ft. 108 sq. in.

6. 9876543 sq. in. to acres.

Ans. 1 A. 91 P. 252 sq. ft. 51 sq. in.

SURVEYORS' SQUARE MEASURE.

275. Surveyors' Square Measure is used by surveyors in computing the area of land.

TABLE.

10,000 square links (sq. li.)	=	1 square chain, sq. ch.
10 square chains . .	=	1 acre, . . A.
640 acres . . .	=	1 square mile, sq. mi.
36 sq. mi. (6 miles square)	=	1 township, Tp.

Also 625 sq. li. = 1 perch; 16 perches = 1 sq. chain; 10 sq. ch. = 1 acre; or 40 perches = 1 rood; 4 roods = 1 acre. The *perch* and *rood* are not so much used as formerly, the contents of land being commonly estimated in square miles, acres, and hundredths.

MENTAL EXERCISES.

How many

1. Square links in 4 sq. ch.?

2. Square chains in 60000 sq. li.?

3. Acres in 60 sq. ch.? 2 sq. mi.?

4. Sq. chains in 65 acres? 42 acres?

5. Square miles in 1280 A.? 2 Tp.?

6. Townships in 72 sq. mi.? 144 sq. mi.?

WRITTEN EXERCISES.

1. Reduce 21 sq. mi. 65 A. 9 sq. ch. to sq. links.

Ans. 1350590000 sq. li.

2. Reduce 1,000,000 sq. li. to acres. *Ans.* 10 A.

3. 2 Tp. 21 sq. mi. 47 A. to sq. ch. *Ans.* 595670 sq. ch.

4. 8700 sq. chains to sq. miles. *Ans.* 1 sq. mi. 230 A.

5. 3 Tp. 250 A. 9 sq. ch. to sq. links. *Ans.* 6937090000.

MEASURES OF VOLUME.

276. A Volume is that which has length, breadth and thickness, or height. A volume is also called a *solid.*

277. A Cube is a volume bounded by six equal squares.

278. All Volumes are *measured* by ascertaining the number of times they contain a *small cube* regarded as a *unit of measure.*

Thus, in the cube in the margin, it will be seen that there are 3 times 3, or 9 cubes upon one surface, and since there are three such layers, there are 3 times 9, or 27 little cubes in all; and since these make up the entire volume, the measure of the cube, called its *contents*, is 27 cubic units.

CUBIC, OR SOLID MEASURE

279. Cubic, or Solid Measure is used in measuring things which have length, breadth, and thickness.

TABLE.

1728 cubic inches (cu. in.)	=	1 cubic foot	cu. ft.
27 cubic feet . .	=	1 cubic yard,	cu. yd.
16 cubic feet . .	=	1 cord foot,	cd. ft.
8 cord feet, or 128 cubic feet . .	=	1 cord of wood,	cd.

I. A *cord of wood*, so named from being originally measured by a *cord*, or *string*, is a pile 8 ft. long, 4 ft. wide, and 4 ft. high. A *cord foot* is a part of this pile 1 ft. long; it equals 16 cubic feet. *See Art.* 358.

II. The *ton* of 40 ft. for *round*, or 50 ft. for *hewn* timber, is seldom used.

MENTAL EXERCISES.

How many

1. Inches in 2 cu. ft.? 4 cu. ft.?
2. Feet in 3 cu. yd.? 5 cd. ft.?
3. Cubic yards in 81 cu. ft.?
4. Feet in 6 cd.? in 4 cd.?
5. Cords in 256 cu. ft.? 72 cd. ft.?
6. Cubic yards in 135 cu. ft.?
7. What is the difference between a 2 inch cube and 2 cubic inches? between a 3 inch cube and 3 cubic inches?

WRITTEN EXERCISES.

Reduce

1. 3 cd. 7 cd. ft. to cubic feet. *Ans.* 496 cu. ft.
2. 7856 cd. ft. to cords. *Ans.* 982 cd.
3. 16 cd. 14 cu. ft. to cubic feet. *Ans.* 2062 cu. ft.
4. 19876 cu. ft. to cords. *Ans.* 155 cd. 36 cu. ft.
5. In 78 cords of wood how many cubic inches?
Ans. 17252352.
6. In 5876543 cubic inches how many cords of wood?
Ans. 26 cd. 72 cu. ft. 1343 cu. in.

MEASURES OF CAPACITY.

280. Measures of Capacity are volumes used to determine the quantity of fluids and many dry substances.

281. Measures of capacity are therefore of two kinds; *Measures of Liquids* and *Measures of Dry Substances.*

282. Liquid Measures are of two kinds,—*Liquid* or *Wine Measure* and *Apothecaries' Fluid Measure.*

LIQUID OR WINE MEASURE.

283. Liquid or **Wine Measure** is used for measuring all kinds of liquids.

TABLE.

4 gills (gi.) . . .	=	1 pint, . .	pt.
2 pints . . .	=	1 quart, . .	qt.
4 quarts . . .	=	1 gallon, . .	gal.
31½ gallons . . .	=	1 barrel, . .	bar.
63 gallons, or 2 bar. .	=	1 hogshead, .	hhd.

I. NAME.—It is called *Wine Measure* because wine was measured by it, while beer was measured by another measure.

II. TERMS.—*Gill* is from Low Latin *gilla*, a *drinking-glass*; *pint* is from the Anglo-Saxon *pyndan*, *to shut in*, *to pen*, or from the Greek *pinto*, *to drink*; *quart* is from the Latin *quartus*, a *fourth*. The derivation of *gallon* is not clear; in the French, a *galon* is a *grocer's box*.

III. UNIT.—The *standard unit* of wine measure is the *gallon*, which contains 231 cubic inches, and will hold a little more than 8⅓ lb. Av. of distilled water. This is called the *Winchester gallon*, from the standard having been formerly kept at Winchester, England.

IV. *Barrels* and *hogsheads* are of variable capacity. The table values are used in estimating the capacity of wells, cisterns, vats, etc. In Massachusetts, the barrel is estimated at 32 gallons. A pint of water weighs

nearly one pound, hence the old adage, "A pint's a pound, the world around."

V. Ale, beer, and milk, were formerly sold by a *gallon* of 282 cu. in., the subdivisions being *quarts* and *pints*. The measure was greater than wine measure, as beer was less costly than wine. This measure is now seldom used.

APOTHECARIES' FLUID MEASURE.

284. Apothecaries' Fluid Measure is used for measuring liquids in preparing medical prescriptions.

TABLE.

60 minims (♏)	. .	=	1 fluidrachm	.	ʒ.
8 fluidrachms	. .	=	1 fluidounce	.	℥.
16 fluidounces	. .	=	1 pint	. .	O.
8 pints	. . .	=	1 gallon	. .	Cong.

I. TERMS.—*Minim* is from the Latin *minimus*, *the least*, the minim being the smallest fluid measure used. Several of the other terms are formed by prefixing *fluid* to the terms of Apothecaries' Weight.

II. SYMBOLS.—*Cong.* is the abbreviation of *congius*, the Latin for gallon. *O.* is the initial of *octarius*, the Latin for *one-eighth*, the pint being one-eighth of a gallon. Drops are indicated in a physician's prescription by *gtt.*, for the Latin *guttæ*.

III. In estimating the quantity of fluids, 45 drops equal about a fluidrachm; a common teaspoon holds about one fluidrachm; a common tablespoon, about ½ a fluidounce; a wineglass about 1½ fluidounces; a common teacup about 4 fluidounces. The minim is equivalent to a drop of water, but the drops of different fluids vary in size according to the tenacity of the liquid.

DRY MEASURE.

285. Dry Measure is used in measuring dry substances, such as grain, fruit, salt, coal, etc.

TABLE.

2 pints (pt.)	. .	=	1 quart,	. .	qt.
8 quarts	. . .	=	1 peck,	. .	pk.
4 pecks	. . .	=	1 bushel,	. .	bu.

I. TERMS.—*Peck* is supposed to be a corruption of *pack*, or to be derived from the French *picotin*, a *peck*.

II. UNIT.—The *unit* of dry measure is the Winchester bushel, formerly used in England, and named from the place where the standard was preserved. Its form is a cylinder, 18½ in. in diameter and 8 in. deep. Its volume is 2150.42 cu. in., and it contains 77.627413 lb. Av. of distilled water. The New York bushel is nearly identical with the imperial bushel of Great Britain, containing 2218.192 cu. in.

III. The *Chaldron*, consisting in some places of 36 bu. and in others of 32 bu., is used in some parts of the United States for measuring coal, but is being discontinued here, as it has been in England. One-half of a peck, or 4 quarts, is called a *dry gallon*.

IV. The *Cental* of 100 lb. is a standard recently recommended by the Boards of Trade in New York, Cincinnati, Chicago, and other large cities, for estimating grain, seeds, etc.

286. The **Weight of a Bushel** of the principal kinds of grain and seeds has been fixed by statute in many of the States, as shown by the following

TABLE.

	Cal.	Conn.	Del.	Ill.	Ind.	Iowa.	Ky.	La.	Me.	Mass.	Mich.	Minn.	Mo.	N. C.	N. H.	N. J.	N. Y.	Ohio.	Or.	Penn.	Vt.	W. T.	Wis.
Barley,	50			48	48	48	48	32		46	48	48	48	48		48	48	48	46	47	46	45	48
Buckwheat,	40	45		40	50	52	52			46	42	42	52	50		50	48		42	48	46	42	52
Clover Seed,				60	60	60	60				60	60	60			64	60	60	60			60	60
Indian Corn,	52	56	56	52	56	56	56	56		56	56	56	52	54		56	58	56	56	56	56	56	56
Oats,	32	28		32	32	35	33½	32	30	30	32	32	35	30	30	30	32	32	34	32	32	36	32
Rye,	54	56		54	56	56	56	32		56	56	56	56			56	56	56	56	56	56	56	56
Timothy Seed				45	45	45	45						45				44						46
Wheat,	60	56	60	60	60	60	60	60		60	60	60	60	60		60	60	60	60	60	60	60	60

MENTAL EXERCISES.

How many

1. Pints in 7 qt.? 45 gi.?
2. Quarts in 12 gal.? 15 gal.?
3. Quarts in 3 pk.? 7 pk.?
4. Gills in 7 pt.? 12 pt.?
5. Pecks in 4 bu.? 9 bu.?
6. Bushels in 20 pk.? 40 pk.?
7. Fluidrachms in 120 ℥?
8. Pints in 64 ʒ? in 5 Cong.?

9. What part of 2 quarts is 2 gills? What part of 3 gallons is 6 pints?

10. At 10 cents a peck, how many bushels of apples can be bought for 12 dollars?

WRITTEN EXERCISES.

How many

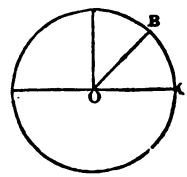
1. Pints in 16 gal. 3 qt. 1 pt. of wine? *Ans.* 135 pt.
2. Gallons in 7635 gills of vinegar?
   Ans. 238 gal. 2 qt. 3 gi.
3. Minims in 5 Cong. 3 O. 12 ʒ. 5 ʒ? *Ans.* 336300 ℥.
4. Cong. in 724475 ℥?
   Ans. 11 Cong. 6 O. 5 ʒ. 2 ʒ. 35 ℥.
5. Pints in 26 bu. 3 pk. 7 qt. 1 pt. of berries?
   Ans. 1727 pt.
6. Bushels in 37891 pints of clover seed? *Ans.* $592\frac{3}{64}$ bu.
7. What cost 5 gal. 3 qt. 1 pt. of vinegar at 3 cents a pint?
   Ans. $1.41.

8. What cost 217 bu. 3 pk. 5 qt. of canary seed at $18\frac{3}{4}$ cents a quart *Ans.* $1307.43\frac{3}{4}$.

CIRCULAR MEASURE.

287. Circular Measure is used to measure angles and directions, latitude and longitude, difference of time, etc.

288. A **Circle** is a plane figure bounded by a curved line, every point of which is equally distant from a point within, called the *centre*.



289. The **Circumference** of a circle is the bounding line; any part of the circumference, as BC, is an *arc*. An arc of one-fourth of the circumference is called a *quadrant*.

290. For the purpose of measuring angles, the circumference is divided into 360 equal parts, called *degrees*; each degree into 60 equal parts, called *minutes*; each minute into 60 equal parts, called *seconds*.

291. Any angle having its vertex at the centre, is measured by the arc included between its sides; thus, the angle COB is measured by the arc BC. A right angle is measured by 90 degrees, or a quadrant; half a right angle, by 45 degrees; etc.

TABLE.

60 seconds (″)	.	.	=	1 minute	′
60 minutes	.	.	=	1 degree	°
30 degrees	.	.	=	1 sign	S.
12 signs, or 360°	.	.	=	1 circumference	C.

I. TERMS.—The term *minute* is from the Latin *minutum*, which signifies a *small part*. The term *second* is an abbreviated expression for *second* minutes, or minutes of the *second order*. *Signs* are used in astronomy as a measure of the zodiac.

II. UNIT.—The *unit* is the *degree*, which is $\frac{1}{360}$ of the circumference of a circle. A *quadrant* is one-fourth of a circumference, or 90°. A *minute* of the earth's circumference is called a *geographic mile*.

III. DIVISIONS.—The divisions of the circumference are not of absolute length; they are merely *equal parts*, used to indicate the size of angles. Thus, a *quadrant*, whether the circle is large or small, measures a *right angle*.

WRITTEN EXERCISES.

Reduce

1. 25′ 30″ to seconds. *Ans.* 1530″.
2. 6000″ to degrees. *Ans.* 1° 40′.
3. 4S. 17° 18′ 42″ to seconds. *Ans.* 494322″.
4. 195600″ to signs. *Ans.* 1S. 24° 20′.
5. 3 quadrants 15° to seconds. *Ans.* 1026000″.
6. If Venus, at a certain time, is 44° 27′ 30″ east of Mercury, how many seconds are they apart?

Ans. 160050″.

TIME.

292. Time is a portion of duration. The *measures* of time are fixed by the revolution of the earth on its axis and around the sun.

TABLE.

60 seconds (sec.) .	=	1 minute, . .	min.
60 minutes . .	=	1 hour, . .	h.
24 hours . .	=	1 day, . .	da.
365 days . .	=	1 common year, . .	yr.
366 days . .	=	1 leap year, .	yr.
100 years . .	=	1 century, .	cen.

ALSO,

7 days . . .	=	1 week,	wk.
4 weeks . . .	=	1 lunar month,	mo.
13 lunar months, 1 d., 6 h., or 12 calendar months,	=	1 year, .	yr.

I. TERMS.—*Second* and *minute* are parts of an hour, corresponding to the parts of a degree in Circular Measure. *Hour* is derived from the Latin *hora*, originally a definite space of time fixed by natural laws; a *day*, derived from the Saxon *daeg*, is the time of the revolution of the earth upon its axis; a *week* is a period of uncertain origin, but which has been used from time immemorial in Eastern countries; a *month*, from Saxon *monadh*, from *mona*, the moon, is the time of one revolution of the moon around the earth; a *year*, from the Saxon *gear*, is the time of the earth's revolution around the sun; a *century* comes from the Latin *centuria*, a collection of a hundred things.

II. UNIT.—The *unit of time* is the *day*; it is determined by the revolution of the earth on its axis. The *Sidereal Day* is the exact time of the revolution of the earth on its axis. The *Solar Day* is the time of the apparent revolution of the sun around the earth. The *Astronomical Day*

is the solar day, beginning and ending at noon. The *Civil Day* is the average length of all the solar days of the year; it begins at 12 o'clock midnight, and consists of two periods of 12 hours each.

THE CALENDAR.

293. **The Calendar** is a division of time into periods adapted to the purposes of civil life.

294. The **Year** is divided into 12 calendar months, three of which constitute a period called a *Season*.

295. The seasons, months, and number of days in each, are given in the following table:

No. of Mo.	Month.	Season.	No. of Days.
1	January,	Winter,	31
2	February,	Winter,	28 or 29
3	March,	Spring,	31
4	April,	Spring,	30
5	May,	Spring,	31
6	June,	Summer,	30
7	July,	Summer,	31
8	August,	Summer,	31
9	September,	Autumn,	30
10	October,	Autumn,	31
11	November,	Autumn,	30
12	December,	Winter,	31

296. The time from any day of one month to any day of another month in the same year is readily found by the following table:

TABLE

SHOWING THE NUMBER OF DAYS FROM ANY DAY OF ONE MONTH TO THE SAME DAY OF ANY OTHER MONTH IN THE SAME YEAR.

From any day of	To the same day of											
	Jan.	Feb.	Mar	Apr	May	J'ne	July	Aug	Sep.	Oct.	Nov	Dec.
January	365	31	59	90	120	151	181	212	243	273	304	334
February	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August	153	184	212	243	273	304	334	365	31	61	92	122
September	122	153	181	212	242	273	303	334	365	30	61	91
October	92	123	151	182	212	243	273	304	335	365	31	61
November	61	92	120	151	181	212	242	273	304	334	365	30
December	31	62	90	121	151	182	212	243	274	304	335	365

METHOD OF USING THE TABLE.—Suppose we wish to find the number of days from March 10th to November 16th. We find March in the vertical column, and November at the top, and at the intersection we find 245, to which adding 6 days we have 251, the number of days required. The table being constructed for February 28 days, the proper allowance must be made for leap year.

I. TERMS.—January is derived from *Janus*, the god of the year, to whom this month was sacred. February is from *februa*, the Roman festival of expiation, celebrated on the 15th of this month. January and February were added to the Roman calendar by Numa, Romulus having previously divided the year into 10 months. March is from *Mars*, the god of war and reputed father of Romulus. It was the first month of the Roman calendar. April is probably from the Latin *aperire*, to open, from the opening of the buds, or the bosom of the earth in producing vegetation. May is from *Maia*, the mother of Mercury, to whom the Romans offered sacrifices on the first day of this month. June is from *Juno*, the sister and wife of Jupiter, to whom it was sacred. July was named by Mark Antony after *Julius* Cæsar, who was born in this month. It was previously called Quintilis. August was named after *Augustus* Cæsar, who entered upon his first consulate in this month. It was formerly called Sextilis, or sixth month. September, October, November, and December, are respectively named from the Latin numerals, *Septem*, *Octo*, *Novem*, and *Decem*, as when the year began in March, they were the seventh, eighth, ninth, and tenth months, as their names indicate.

II. The number of days in each month is easily remembered by the following stanza :—

Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Excepting February alone;
To which we twenty-eight assign,
Till leap year gives it twenty-nine.

MENTAL EXERCISES.

1. How many seconds in 30 minutes? in 25 minutes?
2. How many minutes in 360 seconds? in 3 hours?
3. How many hours in 660 minutes? in 4 days?
4. How many days in 72 hours? in 4 weeks?
5. How many days in 3 common years? in 4 leap years?
6. How many common years in 730 days?
7. Name the months which have 30 days each; those which have 31 days each.

WRITTEN EXERCISES.

1. How many seconds in one day? *Ans.* 86400 sec.
2. How many seconds in one week? *Ans.* 604800 sec.
3. How many minutes in 5 da. 7 h. 45 min.?
   Ans. 7665 min.
4. Reduce 56780 seconds to hours.
   Ans. 15 h. 46 min. 20 sec.

5. In one common year how many hours; how many minutes; how many seconds?

Ans. 8760 h.; 525,600 min.; 31,536,000 sec.

6. How many days from May 8th to October 19th?

Ans. 164 days.

7. How many days from March 28th to December 16th?

Ans. 263 days.

8. How many days from February 21st, 1860, to September 10th, 1860? *Ans.* 202 days.

ADJUSTMENT OF THE CALENDAR.

297. A **True** or **Solar Year** is the exact time in which the earth revolves around the sun. It consists of 365 da. 5h. 48 min. 49.7 sec. Now, since it is inconvenient to reckon the fractional part of a day each year, it is necessary to arrange a correct calendar in which each year may have a whole number of days. This is done by causing some years to consist of 365 days, and others of 366 days. The former are called common years, the latter, Bissextile, or Leap years.

298. The calendar is reckoned according to the following rule:

Rule.—*Every year that is divisible by* 4, *except the centennial years, and every centennial year divisible by* 400, *is a leap year; all the others are common years.*

NOTE.—The centennial years are those whose expressions in figures end in two ciphers.

EXPLANATION.—I. If we reckon 365 days as one year, the time lost in the calendar in one year is 5 h. 48 min. 49.7 sec., and the time lost in four years is 23 h. 15 min. 18.8 sec., that is, *one day*, lacking only 44 min. 41.2 sec.; hence the first error can be corrected by adding *one* day every *four* years, making the year to consist of 366 days.

II. If every fourth year be reckoned as leap year, since we add 44 min., etc., too much, the time *gained* in the calendar in four years is 44 min. 41.2 sec., and in 100 years it will be 18 h. 37 min. 10 sec., that is, *one day*, lacking 5 h. 22 min. 50 sec.; hence the second error may be corrected by deducting one day from each centennial leap year, thus calling each centennial year a common year of 365 days.

III. Again, if every centennial year be reckoned as a common year, since we do not add enough, the time lost in 100 years will be 5 h. 22 min. 50 sec., and in 400 years it will be 21 h. 31 min. 20 sec.; hence the time lost in 400 years will be 1 day lacking 2 h. 28 min. 40 sec., and this error may be rectified by making every 4th centennial year a leap year

In the same way we may make the calendar correct for any number of years.

NOTE.—The reckoning of time by the ancients was very inaccurate. The calendar was reformed by Julius Cæsar, 46 B. C., who made the year to consist of 365¼ days, adding one day every fourth year. In 1582, Pope Gregory corrected the error which resulted from the above correction, by striking out 10 days from the calendar, calling the 5th of October the 15th, and ordaining that henceforth only those centennial years should be leap years which are divisible by 400. This change was soon adopted by most Catholic countries, but Great Britain did not make the change till 1752, when the error amounted to 11 days. Some countries, as Russia, still adhere to the Julian Calendar, their dates being about 12 days behind ours. The dates are distinguished as Old Style and New Style.

MENTAL EXERCISES.

1. How many centuries is it since the birth of Christ?

2. When did the 18th century end and the 19th century begin?

3. How many leap years and how many common years in every century?

4. Which of the following are leap years: 1700? 1760? 1776? 1800? 1876? 1880? 1890? 1900? 2000?

5. My watch ticks 4 times in a second; how many times will it tick in a day?

MISCELLANEOUS TABLES.

299. The following tables are frequently used, the first in counting certain kinds of articles, and the second in the paper trade.

COUNTING.	PAPER.
12 units = 1 dozen.	24 sheets = 1 quire.
12 dozen = 1 gross.	20 quires = 1 ream.
12 gross = 1 great gross.	480 sheets = 1 ream.
20 units = 1 score.	

I. *Two* things of a kind are frequently called a *pair* and *six* a *set*.

II. Paper is sold at retail by sheets, quires, and reams, and at wholesale by reams.

BOOKS.

300. In printing books large sheets of paper are used, which are folded into leaves according to the size of the book. The terms *folio*, *quarto*, *octavo*, etc., as applied to printed books, are based on sheets about 18×24 in., about half the sizes now generally used, and indicate the number of leaves into which such a sheet is folded.

A sheet folded in 2 leaves is called a folio, makes 4 pages.
A sheet folded in 4 " " " a quarto or 4to, makes 8 pages.
A sheet folded in 8 " " " an octavo or 8vo, makes 16 pages.
A sheet folded in 12 " " " a 12mo, makes 24 pages.
A sheet folded in 16 " " " a 16mo, makes 32 "
A sheet folded in 18 " " " an 18mo, makes 36 "
A sheet folded in 24 " " " a 24mo, makes 48 "
etc. etc.

NOTE.—Printing paper is made of many sizes, according to the requirements of the printer. In book printing 24 × 38 inches, called *Double Medium*, is perhaps used most largely.

301. Clerks and copyists are often paid by the *folio* for making copies of legal papers, records, and documents.

72 words make 1 folio, or sheet of common law.
90 " " 1 " " " " chancery.

MENTAL EXERCISES.

1. How many dozens in ½ a gross? In 2½ gross? In ¼ a great gross?

2. How many pairs in 40? Scores in 60? Scores in 70? Sets in 48?

3. How many sheets in 3 quires? In 2½ quires? In ½ a ream? In ¼ ream?

4. How many eggs in 2½ dozen? In half a dozen? In a dozen and a quarter?

5. How many years in 3 score? In 3 score and 10? In 4 score and a half?

6. How many sheets of paper will be required to make a 12mo. book of 360 pages? Of 480 pages?

7. How many sheets will be required to make an octavo book of 320 pages? Of 400 pages?

8. How many octavo books will the paper for a quarto book make, of the same number of pages?

WRITTEN EXERCISES.

1. How many fine black crayons are there in 42 boxes, each containing 1 gross? *Ans.* 6048.

2. Sold 63 boxes of Maynard's writing ink, each box containing 3 dozen bottles; how many gross?

Ans. 15 gross, 9 dozen.

3. What would be the cost of 3240 sheets of foolscap at 36 cents a quire? *Ans.* $48.60

4. A lady copied in one month 795.5 chancery folios at 12¢ per folio; what did she receive? *Ans.* $95.46.

5. A printer used 3 reams 5 quires 19 sheets of paper for printing half-sheet posters; how many did he print, allowing 1 quire to a ream for waste? *Ans.* 3000.

6. How much paper would it require to print 5000 copies of a 12mo book of 424 pages, allowing 1 quire to each ream for waste?

REDUCTION OF COMPOUND NUMBERS.

302. Reduction is the process of changing a number from one denomination to another without altering its value.

303. There are **Two Cases**: *Reduction Descending* and *Reduction Ascending.*

These two cases have been considered in the examples under the tables, but we will present a few more problems under their proper heads.

REDUCTION DESCENDING.

304. Reduction Descending is the process of reducing a number to a lower denomination.

1. Reduce £6 5 s. 3 d. to pence.

SOLUTION.—In 1 pound there are 20 shillings, and in £6 there are 6 times 20 shillings, or 120 shillings; 120 shillings plus 5 shillings are 125 shillings; in 1 shilling there are 12 pence, and in 125 shillings there are 125 times 12 pence, or 1500 pence; 1500 pence plus 3 pence are 1503 d. Therefore, etc.

OPERATION.

£	s.	d.
6	5	3
20		
125 s.		
12		
1503 d., *Ans.*		

Rule.—I. *Multiply the number of the highest denomination given, by the number of units of the next lower denomination which equals one of this higher, and to the product add the number given, if any, of this lower denomination.*

II. *Multiply this result as before, and proceed in the same manner until we arrive at the required denomination.*

WRITTEN EXERCISES.

2. Reduce 15 lb. 4 oz. 3 pwt. 15 gr. to gr. *Ans.* 88407 gr.
3. 17 ℔. 7℥ 5℥ 1℈ 18 gr. to gr. *Ans.* 101618 gr.
4. 3 cable lengths 100 fathoms to feet. *Ans.* 2760 ft.
5. In 18 T. 13 cwt. 75 lb. 14 oz. how many ounces? *Ans.* 598014 oz.

6. How many inches in 1 degree? *Ans.* 4381977.6 in.

7. How many links in 3 miles 20 chains? *Ans.* 26000 li.

8. Reduce 18 A. 25 P. 16 sq. yd. to sq. inches.

Ans. 113908356 sq. in.

9. How many square chains in one township?

Ans. 230400 sq. ch.

10. Reduce 56 cu. yd. 10 cu. ft. 1500 cu. in. to cubic inches. *Ans.* 2631516 cu. in.

11. Reduce Cong. vj. O. vij. f℥xv. f℥iv. ℿxxx. to minims.

Ans. 429870ℿ.

12. A hogshead of molasses contained 98 gallons; required the number of gills. *Ans.* 3136 gi.

13. Between the times of two observations, the planet Mars was found to have passed over 3 S. 25° 40″; how many seconds in this distance? *Ans.* 414040″.

14. Adam died at the age of 930 years; how many seconds old was he? (See Art. 297.) *Ans.* 29,347,944,621 sec.

15. If the pulse beat 75 times a minute, how often does it beat in a week? *Ans.* 756000 times.

16. How many inches from Lancaster to Philadelphia, if it is just 68 miles? *Ans.* 4,308,480 in.

REDUCTION ASCENDING.

305. Reduction Ascending is the process of reducing a number to a higher denomination.

1. In 46743 pence how many pounds?

SOLUTION.—There are 12 pence in 1 shilling, hence in 46743 pence there are as many shillings as 12 is contained times in 46743, which are 3895 s. and 3 d. remaining; there are 20 shillings in 1 pound, hence in 3895 s. there are as many pounds as 20 is contained times in 3895, which are £194 and 15 s. remaining. therefore, in 46743 pence there are £194 15 s. 3d.

OPERATION.

12)46743

20)3895 + 3 d.

£194 + 15 s.

Ans. £194 15 s. 3 d.

Rule.—I. *Divide the given number by the number of units in that denomination which equals one of the next higher.*

II. *Divide the quotient in the same way, and thus proceed until we arrive at the required denomination*

III. *The last quotient and the remainders, if any, will be the result required.*

WRITTEN EXERCISES.

2. Reduce 2579 pence to pounds. *Ans.* £10 14 s. 11 d.

3. Reduce 76942 grains to pounds Troy.
Ans. 13 lb. 4 oz. 5 pwt. 22 gr.

4. How many pounds in 75826 farthings?
Ans. £78 19 s. 8 d. 2 far

5. Reduce 57642 gr. to ℔. *Ans.* 10 ℔. 2℈ 2 gr.

6. Reduce 65784 oz. to tons.
Ans. 2 T. 1 cwt. 11 lb. 8 oz.

7. How many miles in 179234 inches?
Ans. 2 mi. 265 rd. 1 yd. 8 in.

8. In 74325 li. how many miles?
Ans. 9 mi. 23 ch. 25 li.

9. Reduce 743750 cu. in. to cords.
Ans. 3 cd. 46 cu. ft. 710 cu. in.

10. In 1720177 perches how many square miles?
Ans. 16 sq. mi. 511 A. 17 P.

11. Reduce 347236 ♏ to Cong.
Ans. Cong. v. O. v. f℥iij. f℥iij. ♏xvj.

12. In 6434 gi. how many gallons? *Ans.* 201 gal. 2 gi.

13. How many bushels in 2401 pints?
Ans. 37 bu. 2 pk. 1 pt.

14. How many common years in 47,424,496 seconds?
Ans. 1 yr. 183 da. 21 h. 28 min. 16 sec.

15. A ship changed her longitude in a storm 721 geographical miles; what was the difference in longitude?
Ans. 12° 1′.

16. A butter-dealer sold 6476 pounds of butter in a month; how many firkins did he sell? *Ans.* 115 firkins 36 lb.

17. A Pennsylvania miller ground a quantity of buckwheat and took 72 lb. of flour for toll, which was $\frac{1}{10}$ of the whole; how many bushels were there brought to mill if 4 lb. of grain produced 3 lb. of flour? *Ans.* 20 bushels.

MENTAL EXERCISES.

MISCELLANEOUS EXAMPLES.

1. How much will 9 eggs cost, at 20 cents a dozen?
2. How much will 8 pearl buttons cost, at 75 cents a dozen?
3. What will 4 table-spoons cost, at the rate of $11.52 a gross?
4. What will shoe-tacks cost a dozen, at the rate of $2.88 a great gross?
5. What will 8 sheets of paper cost, at 18 cents a quire?
6. What will 12 quires of paper cost, at $4 a ream?
7. What cost ½ a gross of lead-pencils, at 50 cents a dozen?
8. What will 2 reams of sand-paper cost, at 22 cents a quire?
9. If a ream of emery-paper costs $12, what does it cost a quire?
10. What will 4 gross of pens cost, if the wholesale rate is 2 pens for a cent?
11. A vessel was sunk in 8 fathoms of water; how many feet deep was it?
12. How many feet high is a horse which measures 15½ hands high?
13. How much will 3 pecks of potatoes cost, at 80 cents a bushel?
14. When apples sell at 20 cents a peck, how much are they worth a bushel?
15. What will 8 quarts of molasses cost, at 96 cents a gallon?
16. How many vials, holding a gill each, can be filled with half a gallon of alcohol?
17. How many half-pint bottles will 2 gallons of Arnold's writing-fluid fill?
18. If I pay $1.60 for a bushel of blackberries, how much is that a quart?
19. If it takes 6 buttons for 1 shirt, how many shirts will 2 gross of buttons trim?
20. The distance across a city park is 132 paces; how many rods is it?
21. What will 15 pounds of flour cost, at the rate of $4 a hundred weight?
22. A boy bought a bushel of chestnuts for $2.40, and sold them at 12 cents a quart; what did he gain?
23. I bought 80 lb. of flour for $1.50; how much is that a hundred-weight?
24. Albert's horse is 5 feet in height; how many hands high is his horse?
25. A vessel sails 4 leagues an hour; how many hours will it take to sail 120 miles?

WRITTEN EXERCISES.

MISCELLANEOUS EXAMPLES.

1. What will 5 tons of coal cost, at 37½¢ a hundred?
Ans. $37.50.

2. How many guineas in 47 pounds and 5 shillings?
Ans. 45 guineas.

3. How many pounds Avoirdupois in 105000 grains?
Ans. 15 lb.

4. At 6¢ a pound, what will 5 cwt. 75 lb. of sugar cost?
Ans. $34.50.

5. How much will 6 barrels of flour cost, at the rate of 3½ cents a pound? *Ans.* $41.16.

6. How many suits of clothes, containing 6¾ yd. each, can be cut out of 93 yd. of cloth? *Ans.* 13; 5¼ yd. over.

7. How many doses of medicine, each weighing 7 grains, can be made out 1 ℥ 2 ʒ 2 ℈ 11 gr.? *Ans.* 93 doses.

8. How many cannon-balls, each weighing 41 lb. 10⅔ oz., can be made out of a ton of iron? *Ans.* 48 balls.

9. How many times will a wheel, 15 ft. 4 in. in circumference, revolve in going 50 miles? *Ans.* 17217$\frac{9}{22}$.

10. Which is greater, and how much, six dozen dozen or a half a dozen dozen? *Ans.* 1st, by 792.

11. How many kegs, each containing 5 gal. 2 qt. 1 pt., can be filled from a tun (4 hhd.) of wine? *Ans.* 44$\frac{4}{5}$.

12. How many lots of 5 A. 82 P. are there in a field containing 66 A. 24 P.? *Ans.* 12 lots.

13. At 13 cents a pound, how much rice can be bought for $81,250? *Ans.* 312½ tons.

14. How much time will a person lose in 50 years by taking an hour's nap each afternoon? *Ans.* 2 yr. 30$\frac{5}{12}$ da.

15. If a comet pass through an arc of 7° 5′ a day, how long will it be in describing an arc of 270°? *Ans.* 38$\frac{2}{17}$ da.

16. How many minutes longer was January, 1860, than February of the same year? *Ans.* 2880 min.

17. If a physician uses on an average 5℥ 7ʒ 1℈ 4 gr. of drugs daily, how many did he use during February, 1876?
Ans. 14 ℔. 3℥ 6ʒ 1℈ 16 gr.

18. How long will it take to count a million at the rate of 80 per minute, working 12 hours a day? *Ans.* $17\frac{13}{36}$ da.

19. How many half-pint bottles will it take to put up 6 gallons of Arnold's writing fluid? *Ans.* 96.

20. How many more seconds were there in 1876 than in a solar year? (See Art. 297.) *Ans.* 65470.3 sec

21. How many boards, 12 ft. long, will inclose a lot 50 rd long and 27 rd. wide, the fence being 3 boards high?

Ans. $635\frac{1}{4}$.

22. How many ounces of calomel will it take to make 980 pills of 5 grains each? *Ans.* 10℥ 1℥ 2ʒ.

23. If a weekly newspaper (one sheet) has 5600 subscribers, how many reams of paper will it require in a year, not allowing for waste? *Ans.* 606 reams, 13 quires, 8 sheets.

24. How many minutes more in the spring of every common year than in the autumn? *Ans.* 1440 min.

25. How many pages are there in an octavo book, the printing of which requires 20 fully printed sheets? *Ans.* 320.

26. Mr. A's income averages 4 cents a minute; what will it be during the three summer months? *Ans.* $5299.20.

27. If 32,400 steel pens are made by a factory in a day, how many gross will be made in the month of March?

Ans. 6975 gross.

28. If a grocer's weights are $\frac{1}{4}$ of an oz. in a pound below the legal standard, how much does he gain fraudulently from the sale of 2 bags of Rio coffee, 116 lb. each, true weight, at $18\frac{3}{4}$¢ a pound? *Ans.* $\$0.69\frac{1}{21}$.

29. I bought 5 T. 14 cwt. of hay in a stack, but before it was all delivered, 1 T. 3 cwt. 56 lb. were spoiled by the weather. The price originally agreed upon was $88.35; what should I pay for what I received? *Ans.* $70.091.

30. A New Jersey truck-raiser sold 15 bu. 3 pk. 1 qt. of strawberries, at 11¢ a quart, and agreed to take flour in payment at $3\frac{1}{2}$¢ a pound, as far as it would make an exact number of barrels, and the rest to be paid in cash; how many barrels and how much cash did he receive?

Ans. 8 barrels and 67¢.

ADDITION OF COMPOUND NUMBERS.

306. Addition of Compound Numbers is the process of finding the sum of two or more similar compound numbers.

1. Find the sum of £7 6s. 8d.; £5 9s. 3d.; £14 13s. 10d.; and £11 19s. 11d.

SOLUTION.—We write the numbers so that similar units shall stand in the same column, and begin at the right to add. 11d. plus 10d., plus 3d., plus 8d., are 32d., which by reduction we find equals 2s. and 8d.; we write the 8d. in the pence column, and reserve the 2s. to add to the column of shillings: 2s. plus 19s., plus 13s., plus 9s., plus 6s., are 49s., which by reduction we find equals £2 and 9s.; we write the 9s. in the column of shillings, and reserve the £2 to add to the column of pounds; £2 plus £11, plus £14, plus £5, plus £7, equal £39, which we write under the pounds. Hence the following

OPERATION.

£	s.	d.
7	6	8
5	9	3
14	13	10
11	19	11
39	9	8

Rule.—I. *Write the compound numbers so that similar units stand in the same column.*

II. *Begin with the lowest denomination and add each column separately, placing the sum, when less than a unit of the next higher denomination, under the column added.*

III. *When the sum equals one or more units of the next higher denomination, reduce it to this denomination, write the remainder under the column added, and add the quotient obtained by reduction to the next column.*

IV. *Proceed in the same manner with all the columns to the last, under which write the entire sum.*

Proof.—The same as in addition of simple numbers.

NOTE.—In writing, if any places are wanting supply them with a cipher.

WRITTEN EXERCISES.

(2)

£	s.	d.
16	12	5
127	13	7
192	18	10
168	14	11
505	19	9

(3)

£	s.	d.
125	16	7
116	13	9
242	17	5
363	15	8

(4)

£	s.	d.
672	18	11
149	10	9
941	17	10
876	19	8

(5)

lb.	oz.	pwt.	gr.
16	10	18	12
14	9	17	23
15	7	15	20
17	11	14	19
23	8	16	22

(6)

cd.	cu. ft.	cu. in.
18	123	762
25	109	835
37	127	976
21	113	204
36	120	630

(7)

T.	cwt.	lb.	oz.
18	18	68	15
25	12	44	13
52	14	73	9
71	17	92	7
26	15	74	14

(8)

lb.	℥	ʒ	℈	gr.
28	11	7	2	16
25	10	6	1	15
19	9	5	2	3
27	8	3	1	17
24	7	2	1	18

(9)

hhd.	gal.	qt.	pt.
27	36	2	1
38	60	3	0
49	55	1	1
74	27	0	1
107	19	3	0

(10)

lb.	℥	ʒ	℈	gr.
14	10	6	2	14
26	9	3	1	18
37	8	2	2	16
15	0	0	0	2
46	7	5	1	17

11. Find the sum of 16 mi. 100 rd. 3 yd. 2 ft. 7 in., 12 mi. 309 rd. 2 yd. 1 ft. 6 in., 15 mi. 274 rd. 5 yd. 2 ft. 9 in., 18 mi. 227 rd. 4 yd. 1 ft. 8 in. *Ans.* 63 mi. 273 rd. 1 ft.

12. Find the sum of 132 sq. yd. 8 sq. ft. 120 sq. in., 246 sq. yd. 7 sq. ft. 137 sq. in., 546 sq. yd. 3 sq. ft. 129 sq. in., 765 sq. yd. 6 sq. ft. 105 sq. in., 382 sq. yd. 5 sq. ft. 126 sq. in. *Ans.* 2074 sq. yd. 6 sq. ft. 41 sq. in.

13. Find the sum of 16 A. 104 P. 18 sq. yd. 7 sq. ft., 25 A. 116 P. 28 sq. yd. 8 sq. ft., 18 A. 139 P. 17 sq. yd. 6 sq. ft., 27 A. 106 P. 30 sq. yd. 8 sq. ft., 24 A. 155 P. 26 sq. yd. 5 sq. ft. *Ans.* 113 A. 144 P. 1 sq. yd. 7 sq. ft.

SUPPLEMENTARY PROBLEMS.

To be omitted unless otherwise directed.

14. A vintner sold to A 5 hhd. 59 gal. 3qt. 1 pt. of wine, to B 20 hhd. 45 gal. 2qt., to C 39 hhd. 58 gal. 1 pt., and had as much as he sold A remaining; how much had he at first?

Ans. 72 hhd. 34 gal. 1½ qt.

15. What is the sum of 126 yr. 10 mo. 5 wk. 17 hr., 236 yr, 9 mo. 2 wk. 7 da. 18 hr. 41 min., 425 yr. 8 mo. 4 wk. 3 da. 20 hr. 16 min., 198 yr. 7 mo. 6 wk. 19 hr. 52 min., 385 yr. 5 wk. 40 min.?

Ans. 1373 yr. 3 mo. 3 wk. 6 da. 4 h. 29 min.

16. Find the sum of 144 cu. yd. 18 cu. ft. 1329 cu. in., 275 cu. yd. 25 cu. ft. 1076 cu. in., 382 cu. yd. 17 cu. ft. 1521 cu. in., 420 cu. yd. 20 cu. ft. 1507 cu. in., 367 cu. yd. 21 cu. ft. 1478 cu. in.

Ans. 1591 cu. yd. 23 cu. ft. 1722 cu. in.

SUBTRACTION OF COMPOUND NUMBERS.

307. Subtraction of Compound Numbers is the process of finding the difference between two similar compound numbers.

1. From 10 oz. 12 pwt. 20 gr. take 7 oz. 15 pwt. 16 gr.

SOLUTION.—We write the subtrahend under the minuend, placing similar units in the same column, and begin at the lowest denomination to subtract; 16 gr. subtracted from 20 gr. leaves 4 gr. which we write under the grains: 15 pwt. from 12 pwt. we cannot take; we will therefore take 1 oz. from the 10 oz., leaving 9 oz.; 1 oz. equals 20 pwt., which, added to 12 pwt. equals 32 pwt.; 15 pwt. subtracted from 32 pwt. equals 17 pwt., which we write under the pwt.; 7 oz. from 9 oz. (or, since it will give the same result, we may add 1 oz. to 7 oz., and say 8 oz. from 10 oz.) leaves 2 oz. Hence the following

OPERATION.

oz.	pwt.	gr.
10	12	20
7	15	16
2	17	4

Rule.—I. *Write the subtrahend under the minuend, so that similar units stand in the same column.*

II. *Begin with the lowest denomination and subtract each term of the subtrahend from the corresponding term of the minuend.*

III. *If any term of the subtrahend exceeds the corresponding term of the minuend, add to the latter as many units of that denomination as make one of the next higher, and then subtract; add* 1 *also to the next term of the subtrahend before subtracting.*

IV. *Proceed in the same manner with each term to the last.*

Proof.—The same as in the subtraction of simple numbers.

NOTE.—The pupil will notice that the general principle of addition and subtraction is the same as in simple numbers, the difference being in the irregularity of the scale, the units themselves being expressed in the decimal scale.

WRITTEN EXERCISES.

(2)

£	s.	d.	far.
143	11	10	2
115	14	6	3
27	17	3	3

(3)

£	s.	d.	far.
930	17	7	3
246	19	8	1

(4)

lb.	oz.	pwt.	gr.
16	10	16	18
13	11	17	15
2	10	19	3

(5)

lb.	oz.	pwt.	gr.
125	8	14	20
96	9	10	23

(6)

cwt.	lb.	oz.
112	92	12
37	44	13

(7)

T.	cwt.	lb.	oz.
236	13	68	12
127	12	22	10

(8)

hhd.	gal.	qt.	pt.
28	27	0	1
06	30	2	1

(9)

yr.	mo.	wk.	da.	h.
216	10	2	5	16
123	10	3	2	20

(10)

sq. yd.	sq. ft.	sq. in.
226	20	120
134	25	130

(11)

m.	rd.	yd.	ft.	in.
85	260	0	2	8
29	296	5	1	9
55	283	$\frac{1}{2}$	0	11
		$\frac{1}{2}$=1		6
55	283	0	2	5

(12)

A.	P.	sq. yd.	sq. ft.	sq. in.
179	150	16	7	135
134	155	18	8	106
44	154	27$\frac{1}{4}$	8	29
		$\frac{1}{4}$=2		36
44	154	28	1	65

13. A farmer had 200 bu. of wheat and sold 28 bu. 2 pk. 5 qt. 1 pt. to one man, and as much more to another; how much remained? *Ans.* 142 bu. 2 pk. 5 qt.

14. A California miner having 112 lb. of gold, sent his mother 17 lb. 10 oz. 15 pwt. 20 gr., and 3 lb. 16 pwt. less to his father; how much did he retain?

Ans. 79 lb. 3 oz. 4 pwt. 8 gr.

15. Subtract 16 mi. 223 rd. 3 yd. 1 ft. 8 in. from 36 mi. 271 rd. 3 yd. 1 ft. 11 in. *Ans.* 20 mi. 48 rd. 3 in.

SUPPLEMENTARY PROBLEMS.

To be omitted unless otherwise directed.

16. Subtract \$16 57¢ 5$\frac{1}{2}$ mills from \$25 20¢ 7$\frac{1}{2}$ mills, and add 2 eagles and 25$\frac{3}{4}$ dimes to the result. *Ans.* \$31.206$\frac{3}{4}$.

17. Subtract 125 A. 37 P. 29 sq. yd. 4 sq. ft. 140 sq. in. from 240 A. 85 P. 16 sq. yd. 96 sq. in.

Ans. 115 A. 47 P. 16 sq. yd. 6 sq. ft. 136 sq. in.

18. A man bought a cask of molasses, from which there leaked away 11 gal. 3 qt. 1 pt., and then after putting in 12 gal. he found it lacked 16 gal. 1 pt. of containing 63 gal.; how much was in at first. *Ans.* 46 gal. 3 qt.

MULTIPLICATION OF COMPOUND NUMBERS.

308. Multiplication of Compound Numbers is the process of finding the product when the multiplicand is a compound number.

1. Multiply £12 11 s. 7 d. by 8.

OPERATION.

£	s.	d.
12	11	7
		8
100	12	8

SOLUTION.—We write the multiplier under the lowest denomination of the multiplicand, and begin at the right to multiply. 8 times 7 d. are 56 d., which, by reduction, we find equals 4 s. and 8 d.; we write the 8 d. under the pence, and reserve the 4 s. to add to the next product: 8 times 11 s. are 88 s., which, added to the 4 s., equals 92 s., which we find by reduction equals £4 and 12 s.; we write the 12 s. under the shillings, and reserve the £4 to add to the next product; 8 times £12 are £96, plus the £4, equals £100, which we write under the pounds. Hence the following

Rule.—I. *Write the multiplier under the lowest denomination of the multiplicand.*

II. *Begin with the lowest denomination, and multiply each term in succession as in simple numbers, reducing as in addition of compound numbers.*

Proof.—The same as in multiplication of simple numbers

NOTE.—If the multiplier is a large composite number, it will be more convenient to multiply by its factors.

WRITTEN EXERCISES.

(2)

cwt.	lb.	oz.
18	96	9
		5

(3)

lb.	oz.	pwt.	gr.
16	8	15	17
			3

(4)

mo.	da.	h.	min.	sec.
50	10	20	30	40
				7

(5)

£	s.	d.	far.
13	12	9	2
			8

(6)

hhd.	gal.	qt.	pt.
21	35	3	1
			9

(7)

lb.	℥	ʒ	℈	gr.
12	8	7	2	20
				11

8. Multiply 12 L. 2 mi. 232 rd. by 12.

Ans. 154 L. 2 mi. 224 rd.

9. Multiply 23 ch. (36 bu.) 18 bu. 2 pk. 7 qt. 1 pt. by 13.

Ans. 305 ch. 27 bu. 2 pk. 1 qt. 1 pt.

10. A farmer sold 5 loads of hay, each containing 15 cwt. 90 lb.; how much did he sell? *Ans.* 79 cwt. 50 lb.

11. Multiply 13 yr. 10 mo. 3 wk. 5 da. by 15, using the factors of the multiplier. *Ans.* 208 yr. 7 mo. 3 wk. 5 da.

12. If a man walk 17 mi. 300 rd. in each of 21 days, how far will he walk in all? *Ans.* 376 mi. 220 rd.

13. If a farmer raise 60 bu. 3 pk. 6 qt. 1 pt. of grain on one acre, how much can he raise at the same rate on 48 acres?

Ans. 2925 bu. 3pk

14. If a pipe discharges 11 hhd. 40 gal. 2 qt 1 pt. of water in an hour, how much will it discharge in 56 hours?

Ans. 652 hhd. 7 gal.

15. A had 1000 A. of land; he sold B 96 A. 150 P., and C 4 times as much; how much remained? *Ans.* 515 A. 50 P.

16. A farmer raised 4000 bu. of grain; he sold 50 bu. 3 pk. 7 qt. to A, 7 times as much to B, and to C 6 times as much as to A and B together; how much remained?

Ans. 1145 bu. 3 pk.

DIVISION OF COMPOUND NUMBERS.

309. Division of Compound Numbers is the process of finding the quotient when the dividend is a compound number.

310. There are two cases:—

1st. To divide a compound number into equal parts.

2d. To divide one compound number by a similar one.

CASE I.

311. ***To divide a compound number into a number of equal parts.***

1. Divide £103 7 s. 6 d. into 5 equal parts, that is, take $\frac{1}{5}$ of it.

SOLUTION.—We write the divisor at the left of the dividend, and begin at the highest denomination to divide. $\frac{1}{5}$ of £103 is £20 and £3 remaining; £3 equal 60 s., which added to 7 s. equals 67 s.; $\frac{1}{5}$ of 67 s. is 13 s. and 2 s. remaining; 2 s. equal 24 d., which added to 6 d. equals 30 d.; $\frac{1}{5}$ of 30 d. is 6 d. Hence the following

OPERATION.

£	s.	d.
5)103	7	6
20	13	6

Rule.—I. *Begin with the highest denomination of the dividend and divide each term in succession, as in simple numbers.*

II. *When there is a remainder, reduce it to the next lower denomination, add it to the term of that denomination, and divide the result as before.*

III *Proceed in the same manner until all the terms are divided.*

Proof.—The same as in division of simple numbers.

NOTE.—When the divisor is a *large number* and *composite*, the factors being not greater than 12, it is perhaps more convenient to divide by the factors.

WRITTEN EXERCISES.

(2)			(3)				(4)		
£	s.	d.	lb.	oz.	pwt.	gr.	T.	cwt.	lb.
4)61	18	4	6)76	10	14	12	7)112	16	66
15	9	7					16	2	38

(5)			(6)					(7)			
cwt.	lb.	oz.	hhd.	gal.	qt.	pt.	gi.	mi.	rd.	yd.	ft.
8)125	94	12	9)108	42	2	1	2	11)120	313	3	2

8. Five sons share 112 A. 144 P. 24 sq. yd. of land; how much does each receive? *Ans.* 22 A. 92 P. 29 sq. yd.

9. If 9 farmers raise 1137 bu. 3 pk. 4 qt. 1 pt. of grain, what is the average amount raised by each?

Ans. 126 bu. 1 pk. 5 qt. $1\frac{1}{9}$ pt.

10. A miner sends 37 lb. 10 oz. 17 pwt. 16 gr. of gold to his 8 sisters; how much does each receive?

Ans. 4 lb. 8 oz. 17 pwt. 5 gr.

11. A man walked 376 mi. 276 rd. in 22 days; what was the average distance each day? *Ans.* 17 mi. $41\frac{7}{11}$ rd.

12. If 26 casks contain 21 hhd. 11 gal. 2 qt. 1 pt., what is the capacity of each cask? *Ans.* 51 gal. 1 qt. $\frac{17}{26}$ pt.

CASE II.

312. ***To divide one compound number by a similar one.***

1. Divide £26 6 s. 2 d. by £4 15 s. 8 d.

SOLUTION.—£26 6 s. 2 d. equals 6314 pence; £4 15 s. 8 d. equals 1148 pence; and dividing 6314 d. by 1148 d. we obtain a quotient of $5\frac{1}{2}$. From this solution we have the following

OPERATION.

£26 6 s. 2 d. = 6314 d.
£4 15 s. 8 d. = 1148 d.

1148)6314($5\frac{1}{2}$, *Ans.*
6314

Rule.—*Reduce both dividend and divisor to the lowest denomination mentioned in either, and then divide as in simple numbers.*

Proof.—The same as in division of simple numbers.

NOTE.—The division may also be made without reducing to the lowest denomination, and this will be shorter when the quotient is integral.

WRITTEN EXERCISES.

2. Divide £48 7 s. 4 d. by £6 11 d. *Ans.* 8.

3. 69 bu. 3 pk. 6 qt. by 6 bu. 3 pk. 6 qt. *Ans.* $10\frac{8}{87}$.

4. If a man feeds his horse 1 pk. 6 qt. of oats a day, how long will 3 bu. 2 qt. last him? *Ans.* 7 days.

5. How many demijohns, each containing 2 gal. 3 qt. 1 pt., can be filled from a tank holding 71 gal. 3 qt. 1 pt. of wine? *Ans.* 25.

6. A drove of cattle ate 6 T. 15 cwt. 87 lb. of hay in a week; how long will 33 T. 19 cwt. 35 lb. last them?

Ans. 5 weeks.

DIFFERENCE BETWEEN DATES.

CASE I.

313. ***To find the difference of time between two dates.***

1. Washington was born Feb. 22d, 1732, and died Dec. 14th, 1799; what was his age?

SOLUTION.—Dates are expressed in the *number* of the year, the month, and the day; hence the date of his birth is 1732 yr. 2 mo. 22 da., and the date of his death is 1799 yr. 12 mo. 14 da.; and the difference of these two dates will equal his age, which we find to be 67 yr. 9 mo. 22 da.

OPERATION.

	yr.	mo.	da.
	1799	12	14
	1732	2	22
	67	9	22

Rule.—*Write the number of the year, month, and day of the earlier date under the year, month, and day of the later date, and take the difference of the numbers.*

NOTE.—In this method we reckon 30 days to the month; when greater accuracy is required, we reckon the actual number of days in each month. The exact time between two dates is found by the table, Art. 296.

WRITTEN EXERCISES.

2. What is the difference in time from Dec. 12th, 1850, to Jan. 5th, 1860? *Ans.* 9 yr. 23 da.

3. Milton was born Dec. 9th, 1608, and died Nov. 8th, 1675; what was his age? *Ans.* 66 yr. 10 mo. 29 da.

4. Franklin was born Jan. 6th, 1706, and died April 17th, 1790; required his age. *Ans.* 84 yr. 3 mo. 11 da.

5. What is the exact time a note has to run, dated Dec 30th, 1862, and payable Jan. 16th, 1864? *Ans.* 1 yr. 17 da.

6. The Revolution was commenced the 19th of April, 1775, and terminated January 20th, 1783; how long did it continue? *Ans.* 7 yr. 9 mo. 1 da.

7. The Civil War began April 11, 1861, and closed April 9, 1865; how long did it last? *Ans.* 3 yr. 11 mo. 28 da.

8. How many years, months, and days, from your birthday to the present date?

SUPPLEMENTARY PROBLEMS.

To be omitted unless otherwise directed.

9. A man was born the 29th of Feb., 1824, and died the 3d of March, 1860; how many birthdays did he see, and what was his exact age? *Ans.* 10; Age, 36 yr. 4 da.

10. Two men were born on the first day of January, one in the year 1756, the other in 1822; both died 37 years after they were born, upon the 1st of January; required the difference of their ages? *Ans.* 1 day.

11. What is the exact time from 45 min. and 25 sec. past 12 o'clock noon, May 20, 1874, to 24 min. 36 sec. past 4 o'clock P. M., July 4, 1876? *Ans.* 2 yr. 1 mo. 15 da. 3 h. 39 min. 11 sec.

CASE II.

314. ***To find the day of the week upon which any given day of the month will fall, the day of the week of some other date being given.***

NOTE.—A common year ends on the same day it begins, hence each year begins one day later than the preceding year. A year following leap year begins two days later.

1. If the 3d of April, 1876, occur on Monday, on what day of the week will the next 29th of August be?

SOLUTION.—By the table we find the difference of time to be 148 days: dividing by 7, the number of days in a week, we have 21 weeks and 1 day; the 29th of August must therefore be one day after Monday or Tuesday.

OPERATION.

$148 \div 7 = 21, +1$

Rule.—*Find the number of days between the two dates, reduce this number to weeks; the number of days remaining will be the number of days from the given day of the week to the required day.*

NOTE.—In solving the problems after the 2d, the date in a previous problem may be assumed as the starting-point.

WRITTEN EXERCISES.

2. If the year 1776 began on Monday, on what day of the week was the 4th of July following? *Ans.* Thursday.

3. What day of the week was the surrender of Burgoyne, Oct. 17, 1777? *Ans.* Friday.

4. On what day of the week was Daniel Webster born, Jan. 18, 1782? *Ans.* Friday.

5. On what day of the week was the battle of Waterloo fought, June 18, 1815? *Ans.* Sunday.

6. On what day of the week was President Lincoln assassinated, April 14, 1865? *Ans.* Friday.

7. On what day of the week was General Grant inaugurated the first time, March 4, 1869? *Ans.* Thursday.

SUPPLEMENTARY PROBLEMS.

To be omitted unless otherwise directed.

8. The Centennial opened on Wednesday, May 10, 1876; on what day will the next open on the same date? *Ans.* Monday.

9. Suppose a letter should be published purporting to have been written at New Orleans on the day of the battle, and dated Wednesday, Jan. 8, 1815; would it be genuine?

10. Let the pupils now determine, from the above principles, the day of the week upon which they were born.

LATITUDE AND LONGITUDE.

315. The **Latitude** of a place is its distance from the equator, north or south. It is reckoned in degrees, minutes, and seconds, and cannot exceed 90°, or a quadrant.

316. The **Longitude** of a place is its distance, east or west, from a given meridian. It is reckoned in degrees, minutes, and seconds, and cannot exceed 180°, or a semi-circumference.

NOTE.—In adding two longitudes, if their sum exceed 180 degrees, it must be subtracted from 360 degrees for the correct difference of longitude.

MENTAL EXERCISES.

1. From what is latitude reckoned? From what is longitude reckoned?

2. What is the greatest latitude a place may have? What is the greatest longitude a place may have?

3. What places have no latitude? What places have no longitude? What place has neither latitude nor longitude?

4. What is the use of latitude and longitude? Has every place a meridian of longitude?

5. What is the latitude of the equator? the latitude of the poles? the longitude of the poles?

317. From the above principles, to find the difference of latitude or longitude, we have the following rule:

Rule.—*When the latitudes or longitudes are both of the same name, subtract the less from the greater; when they are of different names, take their sum.*

WRITTEN EXERCISES.

1. The latitude of Richmond, Va., is 37° 20′ north, and of Savannah 32° 4′ 56″ north; what is the difference of latitude? *Ans.* 5° 15′ 4″.

2. The latitude of Charleston, S. C., is 32° 46′ 33″ north, and of Quito, 13′ 27″ south; what is the difference of latitude? *Ans.* 33°.

3. The longitude of Portland is 70° 13′ 34″ W., and of Mobile, 88° 1′ 29″ W.; what is the difference of longitude? *Ans.* 17° 47′ 55″.

4. The longitude of New Orleans is 90° W. and of Geneva 6° 9′ 5″ E.; what is the difference of longitude? *Ans.* 96° 9′ 5″.

5. The longitude of St. Paul, Minn., is 95° 4′ 55″ W., and of Berlin 13° 23′ 45″ E.; what is the difference of longitude? *Ans.* 108° 28′ 40″.

6. The longitude of Paris is 2° 20′ E., and of New York 74° 3′ W.; what is the difference of longitude? *Ans.* 76° 23′.

LONGITUDE AND TIME.

318. The earth revolves on its axis from west to east once in 24 hours, and this causes the sun to appear to revolve around the earth from east to west in the same time. Places on the east of a certain point have later time, those on the west earlier time, since the sun appears to those on the east first.

319. The circumference of a circle contains 360°, hence the sun appears to travel through 360° in 24 hours, and in 1 hour it travels $\frac{1}{24}$ of 360°=15°; in 1 minute it travels $\frac{1}{60}$ of 15°=15′, and in 1 second it travels $\frac{1}{60}$ of 15′=15″. Hence the following table:

TABLE OF LONGITUDE AND TIME.

15°	of longitude	=	1 hour of time.
15′	of "	=	1 minute of time.
15″	of "	=	1 second of time.

MENTAL EXERCISES.

1. In what time does the earth revolve on its axis? What part of a revolution does it make in 12 hours? in 6 hours?
2. How many degrees of the earth's surface pass under the sun's rays in 24 h.? in 12 h.? in 4 h.?
3. How many degrees of longitude make a difference of 1 hour in time? 2 hours? 3 hours? 4 hours?
4. In what direction does the earth turn on its axis? In what direction does the sun appear to move?
5. Does the sun appear first to places east or west of a given point?
6. When it is noon with us, is it earlier or later east of us? west of us?
7. When it is noon at Boston, what is the time 15° east of Boston? 15° west? 30° east? 30° west?
8. What difference in longitude makes a difference of 1 hour of time? of 1 minute? of 1 second?
9. What is the difference of longitude between two cities, if the difference of time is 1 hour? 1 h. 30 min.? 2 h.? 2 h. 45 min.?
10. If I start at New York and travel until my watch is 1 h. 30 min. too fast, in what direction and how far do I go?
11. If I start at Chicago and travel until my watch is 2 h. 15 min. too slow, how far and in what direction do I travel?

CASE I.

320. ***To find the difference of time of two places when their difference of longitude is given.***

1. The difference of longitude between two places is 40°; what is their difference of time?

SOLUTION.—Since 15° of longitude correspond to 1 h. of time, and 15′ of longitude to 1 min. of time, $\frac{1}{15}$ of the number of *degrees* and *minutes* will equal the number of *hours* and *minutes* difference in time. Dividing by 15 we have 2 h. 40 min. Hence the following

OPERATION.

15)40° 0′

2 40

Rule.—*Divide the difference of longitude expressed in* ° ′ ″ *by* 15; *the result will be the difference of time in* H MIN. SEC.

WRITTEN EXERCISES.

2. The difference of longitude of two places is 35°; what is their difference of time? *Ans.* 2 h. 20 min.

3. The longitude of Portsmouth is 70° 45′, and of Washington 77° 0′ 15″; required their difference of time.

Ans. 25 min. 1 sec.

4. The long. of New York is 74° 3′ west, and of New Orleans 90° west; required the difference in time.

Ans. 1 h. 3 min. 48 sec.

5. The longitude of Philadelphia is 75° 9′ 5″ west, and of Cincinnati 84° 29′ 31″ west; what is the time at Cincinnati when it is 10 A. M. at Philadelphia?

Ans. 22 min. $38\frac{4}{15}$ sec. past 9 A. M.

6. Vienna is 16° 23′ east long.; what is the time there at 9 A. M. in Philadelphia? *Ans.* 3 h. 6 min. $8\frac{1}{2}$ sec. P. M.

SUPPLEMENTARY PROBLEMS.

To be omitted unless otherwise directed.

7. The long. at Washington is 76° 56′ west of London; what change must we make in our watches in coming from London to Washington? *Ans.* Set back 5 h. 7 min. 44 sec.

8. Paris is about 2° 20′ east longitude; what change would a person have to make in his watch in going from New York to Paris? *Ans.* Set ahead 5 h. 5 min. 32 sec.

9. The longitude of Rome is 12° 27′ east, and San Francisco 122° 26′ 15″ west; what time is it in the latter place when it is 4 P. M. in the former? *Ans.* 27 sec. past 7 A. M.

CASE II.

321. ***To find the difference of longitude of two places when their difference of time is given.***

1. The difference of time between two places is 26 minutes; what is their difference of longitude?

SOLUTION.—Since 1 h. of time corresponds to 15° of longitude, and 1 min. of time to 15′ of longitude, 15 times the number of *hours* and *minutes* difference in time will equal the number of *degrees* and *minutes* difference in longitude. Multiplying by 15 we have 6° 30′. Hence the following

OPERATION.

h.	min.
0	26
	15
6°	30′

Rule.—*Multiply the difference of time expressed in* H. MIN. SEC. *by* 15; *the result will be the difference of longitude in* ° ′ ″.

WRITTEN EXERCISES.

2. The difference of time between Philadelphia and Cincinnati is about 37 min. 20 sec.; what is the difference of longitude? *Ans.* 9° 20′.

3. The time at St. Louis is about 53 minutes earlier than the time at Washington; what is the difference in longitude? *Ans.* 13° 15′.

4. When it is noon at London it is about 7 o'clock, A. M., in Philadelphia; required the difference of longitude. *Ans.* About 75°.

5. In traveling from New York to Cincinnati I find my watch is 41 min. 32 sec. too fast; required the difference of longitude. *Ans.* 10° 23′.

SUPPLEMENTARY PROBLEMS.

To be omitted unless otherwise directed.

6. In coming from San Francisco to Philadelphia I find my watch is 3 h. 9 min. 8⅔ sec. too slow; what is the longitude of San Francisco, that of Phila. being 75° 9′ 5″? *Ans.* 122° 26′ 15″.

7. The longitude of Cambridge, England, is 5′ 21″ east, and the difference of time between it and Cambridge, Mass., is 4 h. 44 min. 50½ sec.; required the longitude of the latter place. *Ans.* 71° 7′ 21″ west.

8. The longitude of New York is 74° 3′ west, and of Jerusalem is 35° 32′ east; when it is 4½ o'clock A. M. at New York, what is the time at Jerusalem? *Ans.* 48 min. 20 sec. past 11 A. M

DENOMINATE FRACTIONS.

322. A **Denominate Fraction** is one in which the unit of the fraction is denominate; as, $\frac{3}{5}$ of a pound.

323. **Denominate Fractions** may be expressed either as *common* fractions or as *decimals*.

REDUCTION OF DENOMINATE FRACTIONS.

324. **Reduction of Denominate Fractions** is the process of changing them from one denomination to another without altering their value.

325. There are **two general cases,** reduction ascending and descending, which, for convenience of operation, are subdivided into several other cases.

REDUCTION DESCENDING.

CASE I.

326. ***To reduce a common denominate fraction to a fraction of a lower denomination.***

1. Reduce $\frac{1}{96}$ of a shilling to farthings.

SOLUTION.—Since there are 12 pence in one shilling, 12 times the number of shillings equals the number of pence; and since there are 4 farthings in 1 penny, 4 times the number of pence equals the number of farthings; hence $\frac{1}{96}$ of a shilling equals $\frac{1}{96}\times\frac{12}{1}\times\frac{4}{1}$ farthings, which by cancelling and multiplying becomes $\frac{1}{2}$ of a farthing. Therefore, etc.

OPERATION.

$\frac{1}{96}\times\frac{12}{1}\times\frac{4}{1}=\frac{1}{2}$ far.

Rule.—*Express the multiplication by the required multipliers, and reduce by cancellation.*

WRITTEN EXERCISES.

Reduce

2. $\frac{1}{160}$ of a bu. to the fraction of a pint. *Ans.* $\frac{2}{5}$.
3. $\frac{1}{1440}$ of an oz. to the fraction of a grain. *Ans.* $\frac{1}{3}$.
4. $\frac{3}{1920}$ of a day to the fraction of a minute. *Ans.* $2\frac{1}{4}$.
5. $\frac{7}{288}$ of a gal. to the fraction of a gill. *Ans.* $\frac{1}{8}$.
6. $\frac{5}{1386}$ of a rod to the fraction of an inch. *Ans.* $\frac{5}{7}$.
7. $\frac{8}{22400}$ of a ton to the fraction of an ounce. *Ans.* $4\frac{2}{7}$.
8. $\frac{6}{64152}$ of a sq. rd. to the fraction of a sq. in. *Ans.* $3\frac{1}{18}$.
9. $\frac{1}{18720}$ of a mile to the fraction of an inch. *Ans.* $3\frac{5}{11}$.

CASE II.

327. ***To reduce a common denominate fraction to integers of lower denomination.***

1. What is the value of $\frac{5}{9}$ of a pound Troy?

SOLUTION.- There are 12 oz. in one pound, hence 12 times the number of pounds equals the number of ounces; 12 times $\frac{5}{9}$ equals $\frac{60}{9}$ or $6\frac{2}{3}$ oz.: there are 20 pwt. in one oz., therefore 20 times the number of oz. equals the number of pwt.; 20 times $\frac{2}{3}$ equals $\frac{40}{3}$, or $13\frac{1}{3}$ pwt., etc.

OPERATION.

$\frac{5}{9}\times 12=\frac{60}{9}=6\frac{2}{3}$ oz.
$\frac{2}{3}\times 20=\frac{40}{3}=13\frac{1}{3}$ pwt.
$\frac{1}{3}\times 24=8$ gr.

SOLUTION 2d.—$\frac{5}{9}$ of a pound equals $\frac{1}{2}$ of 5 lb.; and $\frac{1}{2}$ of 5 lb. we find by dividing is 6 oz. 13 pwt. 8 gr.

OPERATION.

lb.	oz.	pwt.	gr.
9)5	0	0	0
6	13	8	

Rule I.—*Reduce the fraction until we reach an integer and a fraction of a lower denomination, set aside the integer and reduce the fraction as before, and thus continue as far as necessary.*

Rule II.—*Regard the numerator as so many units of the given denomination, and divide by the denominator.*

WRITTEN EXERCISES.

What is the value

2. Of $\frac{5}{6}$ of an ounce? *Ans.* 16 pwt. 16 gr.
3. Of $\frac{4}{5}$ of a bushel? *Ans.* 3 pk. 1 qt. $1\frac{1}{5}$ pt.
4. Of $\frac{3}{8}$ of a mile? *Ans.* 213 rd. 1 yd. $2\frac{1}{2}$ ft.
5. Of $\frac{7}{8}$ of a rod? *Ans.* 4 yd. 2 ft. $5\frac{1}{4}$ in.
6. Of $\frac{9}{10}$ of a mile? *Ans.* 288 rd.
7. Of $\frac{7}{8}$ of a sign? *Ans.* 26° 15′.
8. Of $\frac{5}{7}$ of an acre? *Ans.* 114 P. 8 yd. 5 ft. $113\frac{1}{7}$ in.

CASE III.

328. ***To reduce a denominate decimal to integers of lower denominations.***

1. Reduce .875 gal. to integers of lower denominations.

SOLUTION.—There are 4 quarts in one gallon, therefore 4 times the number of gallons equals the number of quarts; 4 times .875 equals 3 qt. and .5 qt.; there are 2 pints in one quart, therefore 2 times the number of quarts equals the number of pints: 2 times .5 equals 1 pt. Therefore .875 gal. equals 3 qt. 1 pt.

OPERATION

.875
4
3.500
2
1.000

Rule —*Reduce the decimal until we reach an integer and a decimal of a lower denomination, set aside the integer, and reduce the decimal as before, and thus continue as far as necessary.*

WRITTEN EXERCISES.

What is the value

2. Of .825 of a pound Troy? *Ans.* 9 oz. 18 pwt.
3. Of .675 of a rod? *Ans.* 3 yd. $2\frac{11}{80}$ ft.
4. Of .364 of an acre? *Ans.* 58 P. $7\frac{13}{50}$ sq. yd.
5. Of .3275 of a hogshead? *Ans.* 20 gal. 2 qt. $1\frac{3}{50}$ pt.
6. Of .9735 of a bushel? *Ans.* 3 pk. $7\frac{19}{125}$ qt.
7. Of .3218 of a ton of iron? *Ans.* 6 cwt. 43 lb. 9.6 oz.
8. Of 2.1365 of a tun (4 hhd.) of wine?

Ans. 2 tuns, 34 gal. 1 qt. 1 pt+.

REDUCTION ASCENDING.

CASE I.

329. ***To reduce a common denominate fraction to a common fraction of a higher denomination.***

1. Reduce $\frac{1}{2}$ of a farthing to the fraction of a shilling.

SOLUTION.—There are 4 farthings in a penny, therefore $\frac{1}{4}$ of the number of farthings equals the number of pence: there are 12 pence in 1 shilling, therefore $\frac{1}{12}$ of the number of pence equals the number of shillings; hence $\frac{1}{2}$ far. equals $\frac{1}{12}\times\frac{1}{4}\times\frac{1}{2}=\frac{1}{96}$ of a shilling.

OPERATION.
$\frac{1}{2}\times\frac{1}{4}\times\frac{1}{12}=\frac{1}{96}$ s.

Rule.—*Express the division by the required divisors, and reduce by cancellation.*

WRITTEN EXERCISES.

Reduce

2. $\frac{3}{4}$ of a grain to the fraction of an ounce. *Ans.* $\frac{1}{640}$.
3. $\frac{9}{10}$ of a foot to the fraction of a mile. *Ans.* $\frac{9}{17600}$.
4. $\frac{3}{8}$ of a gill to the fraction of a gallon. *Ans.* $\frac{1}{32}$.
5. $\frac{5}{7}$ of an inch to the fraction of a rod. *Ans.* $\frac{5}{1980}$.
6. $\frac{7}{8}$ of a lb. to the fraction of a ton. *Ans.* $\frac{7}{16000}$.
7. $4\frac{2}{7}$ oz. to the fraction of a ton. *Ans.* $\frac{3}{22400}$.
8. What part of a cord of wood is a pile containing 48 cubic feet? *Ans.* $\frac{3}{8}$.

CASE II.

330. ***To reduce a compound number to a common fraction of a higher denomination.***

1. Reduce 3 s. 6 d. 2 far. to the fraction of a pound.

SOLUTION.—By reduction we find 3 s. 6 d. 2 far. equal to 170 far., and also £1 = 960 far.; one farthing is $\frac{1}{960}$ of a pound, and 170 far. equals 170 times $\frac{1}{960} = \frac{170}{960}$, which reduced to its lowest terms, equals $\frac{17}{96}$. Therefore, etc.

OPERATION.

3 s. 6 d. 2 far. = 170 far.
£1 = 960 far.
$\frac{170}{960} = \frac{17}{96}$, *Ans.*

Rule.—*Reduce the number to its lowest denomination, and write under it the number of units of this denomination which make a unit of the required denomination; and then reduce the resulting fraction to its lowest terms.*

WRITTEN EXERCISES.

2. Reduce 3 oz. 8 pwt. 12 gr. to pounds. *Ans.* $\frac{187}{4800}$.
3. What part of 2 bushels is 1 bu. 4 qt.? *Ans.* $\frac{9}{16}$.
4. Of a bar. of beer (36 gal.) is 22 gal. 2 qt.? *Ans.* $\frac{5}{8}$.
5. Of a barrel of wine is 4 gal. 3 qt. 1 pt.? *Ans.* $\frac{13}{84}$.
6. Of 5 ℥. is 23 ℔ 10 gr.? *Ans.* $\frac{1}{16}$.
7. Of 3 years is 3 wk. 6 da. 20 h.? *Ans.* $\frac{187}{6570}$.
8. What part of 4 inches square is 4 square inches? What part of a $2\frac{1}{2}$ inch cube is $2\frac{1}{2}$ cu. in.? *Ans.* $\frac{1}{4}$; $\frac{4}{25}$.

CASE III.

331. ***To reduce a compound number to a decimal of a higher denomination.***

1. Reduce 1 bu. 2 pk. 4 qt. to the decimal of a bushel.

SOLUTION.—There are 8 qt. in 1 pk., hence $\frac{1}{8}$ of the number of quarts equals the number of pecks; $\frac{1}{8}$ of 4 equals .5, which, with 2 pk. equals 2.5 pk.; there are 4 pk. in a bushel, hence $\frac{1}{4}$ of the number of pecks equals the number of bushels; $\frac{1}{4}$ of 2.5 equals .625, which, with 1 bu., equals 1.625 bu.; hence 1 bu. 2 pk. 4 qt. equals 1.625 bu.

OPERATION.

8 | 4
4 | 2.5
1.625 *Ans.*

Rule.—I. *Divide the lowest term by the number of units which equals one of the next higher, and annex the decimal quotient to the integer of the next higher denomination.*

II. *Proceed in a similar manner until the whole is reduced to the required denomination.*

NOTE.—It may also be done by reducing to a common fraction, and the common fraction to a decimal.

WRITTEN EXERCISES.

Reduce

2. 5 oz. 10 pwt. 12 gr. to the decimal of a lb. .

Ans. .4604166 lb. +.

3. 3 pk. 4 qt. 1 pt. to the decimal of a bu.

Ans. .890625 bu.

4. 9℥ 1ʒ 2℈ 8 gr. to the decimal of a ℔. *Ans.* .76875 ℔.

5. 286 rd. 1 yd. 3⅔ in. to the decimal of a mile.

Ans. .894375 mi.

6. What decimal part of 5 gal. is 1 qt. 3⅗ gi.?

Ans. .0725.

7. Reduce 295 rd. 3 yd. 2 ft. 8 in. to the decimal of a mile.

Ans. .92408459 mi. +.

8. Reduce 91 lb. 12⅞ oz. to the decimal of a ton.

Ans. .04590234375 ton.

MENTAL EXERCISES.

MISCELLANEOUS EXAMPLES.

1. How many seconds are there in the circumference of a carriage wheel?

2. A lady put up 50 quart cans of strawberries; how many pecks did she put up?

3. When the sun has seemed to pass over 4½ signs, how many seconds has he appeared to move?

4. How many pages of an octavo volume can be printed on 8 quires of paper?

5. A hostler at a hotel stable fed out 48 half-peck measures of oats; how many bushels did he feed?

6. Sarah picked a peck of cherries, and sold them at 4 cents a pint; how much did she receive for them?

7. A sailor took the sounding, and found the water 75 feet deep; how many fathoms was it?

8. My grandfather's age is "three-score years and ten;" how many years old is he?

9. A merchant buys ¼ dozen hand-saws, at $20 a dozen; how shall he sell them to gain 50 cents apiece?

10. I paid $3.20 for a bushel of huckleberries; how shall I sell them to gain 2 cents a quart?

11. A merchant bought ½ dozen shovels, at $9.60 a dozen; how shall he retail them to gain 20 cents apiece?

12. What must I pay a gross for hair-pins, that I may sell them for 5 cents a dozen, and gain 2 cents a dozen?

13. How many cubic inches in a 2-inch cube? in a 4-inch cube? in a 6-inch cube?

14. In 6 francs how many centimes? How many in 8½ francs? in 10½ francs?

15. How many leap years in every century? At what time did the 19th century begin?

16. What part of a circumference is 90°? 60°? 180°? Number of degrees in a quadrant?

17. A grocer paid $6.40 for a bushel of cranberries; how shall he sell them a quart to gain 2 cents a pint?

18. A huckster woman sold one day a bushel of pea-nuts by the half-pint glass, at 5 cents a glass; how much did she receive?

19. A lady bought 2 pairs of vases, 3 sets of chairs, and a dozen knives and forks; how many individual articles did she buy?

20. If Henry Clay spoke 90 words in a minute, how many words would he speak in an address of one hour?

21. Bought paper collars at 40 cents a box, each box containing a dozen; how much will I gain on each collar by selling them for 5 cents apiece?

WRITTEN EXERCISES.

MISCELLANEOUS PROBLEMS.

1. If 8 barrels of flour cost £24 12 s. 8 d., what will 12 barrels cost at the same rate? *Ans.* £36 19 s.

2. If 7 bales of goods weigh 20 cwt. 75 lb., what will 56 bales of the same size weigh? *Ans.* 166 cwt.

3. What cost 8⅔ cords of wood, at the rate of £11 5 d. for 5¾ cords? *Ans.* £16 12 s. 2⅔ d.

4. What cost 8¾ yd. of cloth, at the rate of $4.50 per yard?
Ans. $39.37½.

5. Multiply 8½ d. by 6.33⅓, and from the product take 2 s. 6 d. *Ans.* 1 s. 11⅙ d.

6. Add .00⅞ sq. yd., .04¾ sq. ft., and .0008 sq. in.
Ans. 18.1808 sq. in

7. Reduce 1 shilling, 11 pence and 1.0384 farthings to the decimal of a guinea. *Ans.* .0923 guinea.

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SUPPLEMENTARY PROBLEMS.

To be omitted unless otherwise directed.

14. How many centals of wheat are equivalent to 1200 bushels? w many centals in 150 bar.? *Ans.* 720; 294.

15. How many bushels of buckwheat in Kentucky are equivalent 520 bu. in Illinois? To 650 bu. in Pennsylvania? (See Art. .) *Ans.* 400; 600.

16. A merchant bought in Connecticut 32 bushels of oats at 2¢ ound. and sold them in New York at 80¢ a bushel; what was his fit? (See Art. 286.) *Ans.* $4.48.

17. If B digs 363 rd. 7 yd. of ditch in 35 wk. 5 da., how long will ake to dig 910 rd. 3¾ yd., working 12 h. a day, 6 da. a week, and k. a month? *Ans.* 22 mo. 1 wk. 3½ da.

18. If a river current carries a raft of lumber at the rate of 4 mi. rd. per hour, how long will it be in carrying it a distance of 309 es? *Ans.* 2 da. 16 h.

19. Mr. Owen sold 15 bu. 3 pk. 4 qt. of apples at $2.75 a bushel, took his pay in flour at 3½¢ a pound, receiving only an exact nber of barrels, and in sugar at 12¢ a pound for what remained; v many barrels of flour and how many pounds of sugar did he ive? *Ans.* 6 barrels; 20$\frac{77}{96}$ lb.

20. Two men start from different places on the equator, and vel toward each other till they meet; on comparing their watches h the time of the place of meeting, it is found that the first is 45 utes slow and the second 1 h. 15 min. fast; how far apart were points at which they started, and in what direction did each vel? *Ans.* 2074.8 mi.

21. The distance from a certain toll-gate east to a tavern is 3$\frac{4}{7}$ es; from the toll-gate west to a school-house is 45$\frac{1}{2}$ rods; half y between the tavern and the school-house is a creek 100 yards e; how far from the toll-gate to the middle of the creek; to the ther bank of the creek?

Ans. 1 mi. 195 rd. 2 yd. 2 ft. 9 in.; 1 mi. 204 rd. 3 yd. 1 ft. 3 in.

22. A balloon started from Paris with despatches for Tours, and hted near Bourges, 119 mi. 266.66⅔ rods from Paris. Its actual te was 1½ times this distance, which it made at the rate of 51 mi. d. an hour. Starting at 4 A. M., when did it alight?

Ans. 6 h. 48 min. 21$\frac{3}{11}$ sec. A. M.

SECTION VII.

PRACTICAL MEASUREMENTS.

332. The **Applications of Measures** to the farm, the household, the mechanic arts, etc., are so extensive that we now present a distinct treatment of the subject.

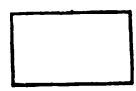
333. These **Practical Measurements** include **Measures** of Surface, Measures of Volume, Measures of Capacity, and Comparison of Weights and of Money.

MEASURES OF SURFACE.

334. A **Surface** is that which has length and breadth without thickness.

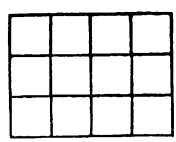
THE RECTANGLE.

335. A **Rectangle** is a plane surface having four sides and four right angles. A slate, a door, the sides of a room, etc., are examples of rectangles.



336. A **Rectangle** has two *dimensions*, length and breadth. A *Square* is a rectangle in which the sides are all equal.

337. The **Area** of a rectangle is the surface included within its sides. It is expressed by the number of times it contains a small square as a *unit of measure*.



Rule I.—*To find the area of a square or rectangle, multiply its length by its breadth.*

For, in the rectangle above, the whole number of little squares is equal to the number in each row multiplied by the number of rows, which is equal to the number of linear units in the length multiplied by the number in the breadth.

Rule II.—*To find either side of a square or rectangle, divide the area by the other side.*

NOTES.—1. The sides multiplied must be of the *same denomination*, and the product will be *square units* of that denomination, which may be reduced, if necessary, to higher denominations.

2. In dividing, the *linear unit* of the side must be of the same name as the *square unit* of the area, and the quotient will be linear units of the same denomination.

WRITTEN EXERCISES.

1. How many square feet in a floor 32 ft. long by 21 ft. wide? how many square yards?

SOLUTION.—To find the area, we multiply the length by the breadth, and we have $32 \times 21 = 672$ sq. ft.; reducing this to square yards, we have $74\frac{2}{3}$ sq. yd.

2. How many square yards in the surface of a blackboard 27 ft. long by 4 ft. wide? *Ans.* 12 sq. yd.

3. How many square yards in a garden 215 ft. long by 109 ft. wide? *Ans.* $2603\frac{8}{9}$ sq. yd.

4. What is the width of a room 25 feet long, whose floor contains 500 sq. ft.? *Ans.* 20 ft.

5. A rectangle contains $466\frac{1}{8}$ sq. ft., and one side is 16 ft 6 in. long; how long is the other side? *Ans.* 28 ft. 3 in.

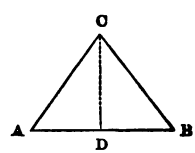
6. How many square feet in the sides of a room 18 ft long, 14 ft. 6 in. wide, and 9 ft. 6 in. high? *Ans.* $617\frac{1}{2}$.

7. A certain box is 3 ft. 6 in. long, 2 ft. 3 in. wide, and 1 ft. 4 in. high; how many square feet in its surface?

Ans. 31 sq. ft. 12 sq. in.

8. What is the surface of a cubical box, each of whose dimensions is 1 ft. 6 in.? *Ans.* $13\frac{1}{2}$ sq. ft.

THE TRIANGLE.



338. A **Triangle** is a plane surface having three sides and three angles; as, ABC.

339. The **Base** is the side upon which it seems to stand, as AB. The **Altitude** is a line perpendicular to the base, drawn from the angle opposite; as, CD.

340. A triangle which has its three sides equal is called *equilateral;* when two sides are equal it is called *isosceles;* when its sides are unequal it is called *scalene.*

Rule I.—*To find the area of a triangle, multiply the base by one-half of the altitude.*

Rule II.—*To find the base or altitude of a triangle, divide the area by one-half the other dimension.*

WRITTEN EXERCISES

1. What is the area of a triangle whose base is 25 inches and altitude 18 inches?

SOLUTION.—To find the area, we multiply the base by one-half the altitude; $25 \times 9 = 225$; hence the area is 225 sq. in.

2. How many square feet in a triangle whose base is 18 ft. 6 in. and altitude 9 ft. 9 in.? *Ans.* 90 sq. ft. 27 sq. in.

3. What is the area of the gable end of a house 29 ft. wide the ridge being 12 ft. higher than the top of the wall?

Ans. 174 sq. ft.

4. The area of a triangular bed of flowers is 25 sq. ft., and its base 10 ft.; what is the altitude? *Ans.* 5 ft.

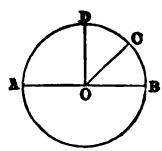
5. The area of a triangular lot is 250 square yards, and its base is 250 ft.; what is its altitude? *Ans.* 18 ft.

6. The area of the gable of a house is 378 sq. ft., the base being 14 yards; what is the height of the ridge?

Ans. 18 ft.

THE CIRCLE.

341. A **Circle** is a plane figure bounded by a curved line, every point of which is equally distant from a point within, called the *centre*.



342. The **Circumference** of a circle is the bounding line; any part of the circumference, as BC, is an *Arc*. An arc of one-fourth of the circumference is called a *Quadrant*.

343. The **Diameter** is a line passing through the centre and terminating in the circumference; as, AB. The *Radius* is a line drawn from the centre to the circumference; as, OD.

Rule I.—*To find the circumference of a circle, multiply the diameter by* 3.1416.

Rule II.—*To find the diameter of a circle, multiply the circumference by* .3183.

Rule III.—*To find the area of a circle, multiply the circumference by one-fourth of the diameter, or multiply the square of the radius by* 3.1416.

WRITTEN EXERCISES.

1. The diameter of a circle is $12\frac{1}{2}$ feet; what is its circumference?

SOLUTION.—To find the circumference, we multiply the diameter by 3.1416; $3.1416 \times 12\frac{1}{2}$ equals 39.27; hence the circumference equals 39.27 ft.

2. What is the circumference of the planet Venus, its diameter being about 7800 miles? *Ans.* 24504.48 miles.

3. The distance round a circular pond is 500 feet; what is the distance across the pond? *Ans.* 159.15 ft.

4. How many times will a carriage wheel 4 ft. 6 in. in circumference revolve in driving 10 miles? *Ans.* $11733\frac{1}{3}$.

5. I have a circular flower-bed 50 feet in circumference; what is the area of the bed? *Ans.* 198 sq. ft. 135 sq. in.

6. A cow is fastened to a stake by a rope 16 feet long; what space can she graze over? *Ans.* 89.36+ sq. yd.

SUPPLEMENTARY PROBLEMS.

To be omitted unless otherwise directed.

7. If the equatorial diameter of the earth is is 7925.75 miles, what are its circumference and the length of a degree of longitude at the equator? *Ans.* 24899.536+ miles; 69.16+miles.

8. A circular flower-bed being divided into four equal parts by lines drawn from the centre, one section was planted with tulips; what was the area of the tulip-bed, its outer edge being 7 feet?

Ans. 15.5967 sq. ft.

MEASUREMENT OF LAND.

344. The **Unit of Measure** of land is the *Acre*, which is sometimes divided into *square rods* and sometimes into *square chains*. Hundredths of an acre are also frequently used.

Government lands are divided by parallels and meridians into *townships*, which contain 36 square miles or *sections*, and each section is sub-

divided into *quarter-sections*. Hence, 640 acres make a *section*, and 160 acres a *quarter-section*. The quarter-sections are still further subdivided into *half-quarter-sections*, *quarter-quarter-sections*, and *lots*. Lots are often of irregular form on account of natural boundaries, but contain, as near as may be, a quarter-quarter-section.

NOTE.—The pupil will remember that *rods* multiplied by *rods* give *square rods*, *chains* by *chains* give *square chains*; also, that 1 acre = 10 square chains or 160 square rods.

WRITTEN EXERCISES.

1. How many square rods in a grass plat 65 ft. long and 15 ft. wide?

SOLUTION.—The area equals 65×15, or 975 sq. ft.; reducing to square rods, we have $3\frac{111}{121}$ sq. rd.

2. How many acres in a rectangular meadow 725 rods long and 400 rods wide? *Ans.* 1812 A. 80 P.

3. What is the value of a farm 208.7 rods long and 120 rods wide, at $\$81\frac{3}{4}$ an acre? *Ans.* $\$12795.91\frac{1}{2}$.

4. Mr. A bought 64 A. 116 P. of land for \$3.50 per square rod, and sold it for \$3.75 per square rod; what did he gain?

Ans. \$2589.

5. A rectangular pond is 200 rd. 17 yd. long, and 150 rd. 15 yd. wide; required its area.

Ans. 193 A. 137 P. $15\frac{3}{4}$ sq. yd.

6. I have a field 16.5 ch. long and 9.75 ch. wide; how much land does it contain? *Ans.* 16 A. 14 P.

7. Mr. Wilson's farm contains 163 A. 3 ch., and its length is 71 ch.; how many rods of fence would be required to surround it? *Ans.* 752 rd.

SUPPLEMENTARY PROBLEMS.

To be omitted unless othermise directed.

8. If a township is equally divided among 480 families, how many acres does each family receive, and what part of a section does each receive? *Ans.* 48 acres; $\frac{3}{40}$ of a section.

9. How many rails are required to fence a quarter-quarter-section, the fence being 5 rails high, and each rail 8 ft. long; and what will be the cost at \$35 per thousand rails? *Ans.* 3300 rails; \$115.50.

10. A field 80 rods long contains 15 acres, while another field of the same width contains 9 acres; what is the length of the latter field? *Ans.* 48 rods.

NOTE.—In Ex. 9, the area is 6400 sq. rd., and each side is 80 rd.

11. How much less will it cost to fence a field 72 rods square than a rectangular field 3 times as long and $\frac{1}{3}$ as wide, if fencing cost $2.50 a rod? *Ans.* $480.

12. A mechanic having a lot of ground 50 rods square, planted 3 acres with corn, 200 square rods with vegetables, 15 rods square with flowers, and the remainder he kept to pasture his cow; how much of the lot was pasture?

Ans. 9 A. 155 P.

COST OF ARTIFICERS' WORK.

345. By **Artificers' Work** we mean plastering, painting, papering, paving, stone-cutting, etc.

346. Plastering, painting, papering, paving, and ceiling are estimated by the *square foot* or *square yard.* Roofing, flooring, partitioning, slating, etc., generally by the *square,* which consists of 100 *square feet,* but sometimes by the square foot or yard.

347. **Shingles,** which commonly measure 18 in. by 4 in., are estimated by the *thousand* or *bundle.* 1000 are generally allowed to a *square* of 100 sq. ft.

WRITTEN EXERCISES.

1. What will be the expense of paving a sidewalk 303 ft. long and $7\frac{1}{2}$ ft. wide, at $2.25 per square yard?

SOLUTION.—The area equals $303 \times 7\frac{1}{2}$, or $2272\frac{1}{2}$ sq. ft., which equals $252\frac{1}{2}$ sq. yd.; hence the cost is $\$2.25 \times 252\frac{1}{2}$, or $\$568.12\frac{1}{2}$.

2. What will it cost to plaster a school-room 40 ft. long, 20 ft. wide, and 10 ft. high, at $0.36 a square yard?

Ans. $80.

3. What is the cost of wainscoting a room 28 ft. long by 15 ft. 4 in. wide, to a height of 4 ft. 3 in. at $0.45 per square yard? *Ans.* $18.41\frac{2}{3}$.

4. What is the cost of slating a roof 52 ft. 10 in. long, each side being 20 ft. wide, at $15.25 per square?

Ans. $322.28\frac{1}{2}$.

5. A frame house is 50 ft. long, 28 ft. wide, and 35 ft. high; what will be the expense of outside painting at $12.25 per square? *Ans.* $668.85.

SUPPLEMENTARY PROBLEMS.

To be omitted unless otherwise directed

6. What will it cost to shingle a roof 64 ft. long and 32 feet from eaves to ridge, the first course along the eaves being double, at $14.87½ a thousand? *Ans.* $614.992.

7. What will it cost to paper a room 40 ft. long, 32 ft. 4 in. wide, and 15½ ft. high, allowing 815 sq. ft. for doors, windows and washboards, at 25¢ per sq. ft.; ceiling not included? *Ans.* $356.83⅓.

8. A cistern 7 ft. 5 in. long, 4 ft. 6 in. wide, and 6 ft. 3 in. deep, is to be lined with zinc costing 12¢ a pound, allowing 5 lb. to the square foot; what will be the expense? *Ans.* $109.40.

CARPETING AND PAPERING.

348. In **Carpeting** we take into consideration the width of the carpet, the allowance for matching the figures, and whether the strips run lengthwise or crosswise.

To match the figures we must often turn under or cut off one of the ends. When an exact number of strips is a little too wide for the room, one breadth is turned under.

Rule.—*Find the number of strips required, and multiply the number of yards in each strip by the number of strips.*

WRITTEN EXERCISES.

1. How many yards of carpet, 1 yd. wide, will be required to carpet a floor 18 ft. 8 in. by 14 ft. 9 in., running lengthwise.

SOLUTION.—The room is nearly 5 yd. wide, hence it will require 5 strips, or breadths, each 18⅔ ft. long; and it will take 18⅔ ft. × 5 = 93⅓ ft., or 31⅓ yds.

2. A lady wishes to cover her sitting-room, 31 ft. long and 15 ft. wide, with matting 2½ ft. wide, no allowance required for matching; how many yards will it take, running lengthwise? How many running crosswise? *Ans.* 63 yds.; 65 yds.

3. How many yards of Brussels carpet, ¾ yd. wide, will it take to carpet a parlor 26 ft. long by 15 ft. wide, the strips running lengthwise, the matching of figures requiring 6 in. to be cut off each strip except the first? *Ans.* 61⅔ yds.

4. Miss Hartman wishes to carpet (lengthwise) a room 18 ft. long by 14 ft. 6 in. wide, with Brussels carpet $\frac{3}{4}$ of a yard wide, at $1.25 a yard; what will it cost, 2 yards waste for matching? *Ans.* $62.50.

SUPPLEMENTARY PROBLEMS.

To be omitted unless otherwise directed.

5. How many rolls of paper 8 yds. long and 20 in. wide, will cover the walls and ceiling of a room 30 ft. long, $22\frac{1}{2}$ ft. wide and 10 ft. 8 in. high, deducting 142 sq. ft. for openings? *Ans.* $41\frac{13}{16}$ rolls.

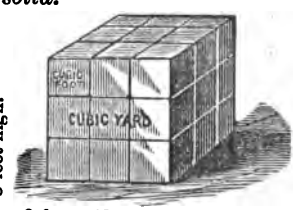
6. What will be the cost of papering the above room at $2.40 a roll, putting also a gilt moulding around the top of the walls, at 12 cents a foot? *Ans.* $111.78.

7. A room contained 3 windows, which were curtained with brocatelle $\frac{1}{2}$ of a yard wide; 10 yards were required for each window @ $1.50, and the curtains were lined with silk $\frac{5}{8}$ of a yard wide @ $.87$\frac{1}{2}$; how many yards of silk were required, and what was the whole cost of the curtains? *Ans.* 24 yd.; $66.

MEASURES OF VOLUME.

349. A Volume is that which has length, breadth, and thickness or height. These three elements are called *dimensions*. A volume is also called a *solid*.

350. A Rectangular Volume or **Solid** is a volume bounded by six rectangles. The bounding rectangles are called *faces*. Cellars, boxes, rooms, etc., are examples of rectangular volumes.



351. A Cube is a volume bounded by six equal squares. Or, a cube is a rectangular volume whose faces are all equal.

352. By the **Contents** or **Solidity** of a volume we mean the amount of space it contains. The contents are expressed by the number of times it contains a *cube* as a *unit of measure*.

Rule I.—*To find the contents of a cube or rectangular volume, take the product of its length, breadth, and height.*

For, in the volume above, the number of cubic units on the base equals the length multiplied by the breadth, or $3 \times 3 = 9$, and the whole number

of cubic units equals the number on the base multiplied by the number of layers of these cubes, or $9 \times 3 = 27$; hence the whole number of cubes, or the contents, equals the product of the length, breadth, and height.

Rule II.—*To find either dimension, divide the contents by the product of the other two dimensions.*

WRITTEN EXERCISES.

1. What are the contents of a room 18 ft. long, 14 ft. wide nd 10 ft. high?

SOLUTION.—To find the contents, we multiply the length, breadth, and height together, and we have $18 \times 14 \times 10 = 2520$ cu. ft.; reducing this to cubic yards, we have 93 cu. yd. 9 cu. ft.

2. What are the solid contents of a cube whose edge measures 1 yd. 1 ft.? *Ans.* 2 cu. yd. 10 cu. ft.

3. A cistern 9 ft. square contains 405 cubic feet; what is its depth? *Ans.* 5 ft.

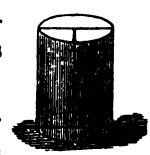
4. How many cubic inches in a rectangular block of marble 6 ft. long, 4 ft. wide, and $2\frac{1}{2}$ ft. thick? *Ans.* 103680.

5. How many cubic yards of air in a room 25 ft. long, 12 ft 6 in. wide, and $9\frac{1}{2}$ ft. high? *Ans.* $109\frac{19}{108}$ cu. yd.

6. A pile of bricks contains 125 cubic yards, and is 13 ft. 6 in. wide, and 8 ft. 4 in. high; what is its length? *Ans.* 30 ft.

7. How much earth will be dug out of a cellar 72 ft. long, 48 ft. wide, and 7 ft. 3 in. deep? *Ans.* 928 cu. yd.

THE CYLINDER.



353. A **Cylinder** is a round body of uniform size, with equal and parallel circles for its ends. The two circular ends are called *bases*.

354. The **Altitude** of a cylinder is the distance from the centre of one base to the centre of the other.

355. The **Convex Surface** of a cylinder is the surface of the curved part.

Rule I.—*To find the convex surface of a cylinder, multiply the circumference of the base by the altitude.*

Rule II.—*To find the contents of a cylinder, multiply the area of the base by the altitude.*

WRITTEN EXERCISES.

1. What is the convex surface of a cylinder, the diameter of whose base is 8 inches and whose altitude is 12 inches?

SOLUTION.—The circumference of the base equals 8×3.1416, which is 25.1328 inches; multiplying by the altitude, 12, we have 301.5936 square inches, the convex surface.

2. I have a log 18 ft. long and 20 inches in diameter; how many square feet of bark on the log? *Ans.* 94.248 sq. ft.

3. A well is 10 feet deep, and 3 feet in diameter; how many cubic feet does it contain? *Ans.* 70.686 cu. ft.

4. What is the cost of digging a well 15 ft. deep and 9 ft. in circumference, at \$.62½ a cubic yard? *Ans.* \$2.24—.

5. How much zinc will it take to line the sides of a cistern 8 ft. in diameter and 8¼ feet deep? *Ans.* 23.0384 sq. yd.

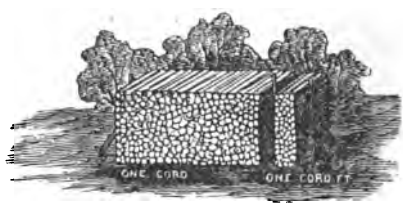
6. Dr. Hiestand put in his house a cistern, 10 ft. in diameter and 4 ft. 6 in. high; how many cubic feet of water did it hold? *Ans.* 353.43 cu. ft.

WOOD MEASURE.

356. The **Measure of Wood** is the *cord*, which is divided into *cord feet*, etc.

357. A **Cord** of wood is a pile 8 feet long, 4 feet wide, and 4 feet high. It contains 8 cord feet, or 128 cubic feet.

358. A **Cord Foot** is a part of this pile 1 foot long. It is thus 1 foot long, 4 feet wide, and 4 feet high, and contains 16 cubic feet.



Rule.—*To find the number of cords in a pile of wood, find the number of cubic feet and reduce to cord feet and cords.*

WRITTEN EXERCISES.

1. How many cords in a pile of wood 28 ft. long, 10 ft. high, and 10 ft. wide?

SOLUTION.—The number of cubic feet equals 28×10×10, which equals 2800; dividing by 16, to reduce this to cord feet, we have 175 cord feet; dividing by 8 to reduce this to cords, we have 21 cd. 7 cd. ft.

2. How many cords in a pile of wood 96 ft. long, 12 ft. wide, and 8 ft. high? *Ans.* 72 cords.

3. A load of wood containing exactly 1 cord, is 5 ft. 4 in. wide, and 3 ft. 9 in. high; what is its length? *Ans.* 6⅔ ft.

4. What is the height of a pile of wood containing 27½ cords, if it is 75 ft. long and 10 ft. wide? *Ans.* 4.736 ft.

5. What will be the cost of the wood that can be piled in a shed 20 ft. long, 10 ft. wide, and 8 ft. high, at $4.75 a cord? *Ans.* $59.37½.

BOARDS AND TIMBER.

359. Boards and **Timber** are usually estimated in what are called *board feet*, instead of in cubic feet.

360. A Board Foot is 1 foot long, 1 foot wide, and 1 inch thick. A *cubic foot*, therefore, contains 12 *board feet*. Hence, board feet may be reduced to cubic feet by dividing by 12, and cubic feet to board feet by multiplying by 12.

A *standard board*, in commerce, is 1 inch thick, and its contents in board feet are the product of its length and breadth in feet. *Board feet* are usually known as *square feet*. Boards are quoted by the *hundred* or the *thousand*, meaning a *hundred square feet*, or a *thousand square feet*. *Round* timber, as masts, etc., is estimated in cubic feet; *hewn* timber, as beams, etc., either in board or cubic feet; *lumber* and *sawed* timber, as planks, scantling, joists, etc., in board feet.

Rule I.—*To find the contents of a board, multiply the length in feet by the width in inches, and divide the product by* 12.

Rule II.—*To find the contents of a plank, joist, etc., multiply the length in feet by the width and thickness in inches, and divide the product by* 12.

NOTES.—1. If one of the dimensions is inches and the other two are feet, the product will be *board feet*.

2. When a board tapers regularly, the length must be multiplied by the *mean width*, which is half the sum of the two ends.

WRITTEN EXERCISES.

1. What are the contents of a board 14 feet long and 9 inches wide?

SOLUTION.—Multiplying the length in feet by the width in inches, we have $14 \times 9 = 126$; and dividing by 12, we have 10½ board feet, or *square feet*.

2. What are the contents of a board 16 feet long and $1\frac{1}{2}$ ft. wide? *Ans.* 24 sq. ft.

3. Required the contents of a board 20 ft. long, the ends being 18 and 14 inches respectively. *Ans.* $26\frac{2}{3}$ sq. ft.

4. How many square feet in 14 planks 16 ft. long, 18 inches wide, and 4 inches thick? *Ans.* 1344 sq. ft.

5. How many square feet in a stick of timber 40 feet long, 14 inches wide, and 9 inches thick? *Ans.* 420 sq. ft.

6. What must be the width of a board 6 ft. 4 in. long that it may contain $9\frac{1}{2}$ square feet? *Ans.* 18 inches.

7. How many square feet of inch boards will it require to make an enclosed box 3 ft. by 2 ft. 6 in. and 18 in. high on the outside, allowing for the overlapping of the boards? *Ans.* $29\frac{1}{3}$ sq. ft.

SUPPLEMENTARY PROBLEMS.

To be omitted unless otherwise directed.

8. What is the cost of 40 boards 12 ft. long, 15 in. wide, at \$2.75 per hundred square feet? *Ans.* \$16.50.

9. What is the cost of 9 pieces of scantling 4 in. by 5 in. and 10 ft. long at \$8.75 per thousand square feet? *Ans.* \$1.31¼.

10. What is the cost of flooring a three-story house, the floors being 56 ft. by 32 ft. and the plank $1\frac{1}{2}$ inches thick, at \$33 per thousand? *Ans.* \$266.112.

11. I wish to fence a field 36 rd. long and 18 rd. wide, the posts to be set 9 ft. apart, the boards to be 18 ft. long, and 11 inches wide, the fence being 3 boards high; the posts cost \$30 per C, and the boards \$15.50 per M, and it required 2 men 3 days, at \$3.50 each a day, to build the fence; required the number of posts, the amount of lumber, and the whole cost. *Ans.* 198 posts; $4900\frac{1}{2}$ sq. ft.; \$156.357.

MASONRY, BRICKWORK, ETC.

361. Masonry is usually estimated by the *perch* and the *cubic foot;* sometimes by the *square foot* or the *square yard.*

362. A Perch *of stone* or of *masonry* is $16\frac{1}{2}$ ft. long, $1\frac{1}{2}$ ft. wide, and 1 ft. high; it contains $24\frac{3}{4}$ cubic feet, but when stone is built into a wall, 22 cubic feet make a perch, $2\frac{3}{4}$ cu. ft. being allowed for mortar and filling.

363. Excavations and **Embankments** are estimated by the *cubic yard.* A cubic yard of earth is called a *load.*

364. Brickwork is generally estimated by the *thousand bricks,* but sometimes in cubic feet.

In estimating *labor,* bricklayers and masons measure the length of the wall on the outside. The corners are thus measured twice, but this is considered an allowance for the greater difficulty of building them. No allowance is made for windows and doors, except by special contract, in which case it is customary to allow one-half the space actually required. In estimating *material,* allowance is made for doors, windows and corners.

The average size of bricks is 8 in.$\times$4$\times$2, but Phila. and Baltimore bricks are $8\frac{1}{4}$ in.$\times 4\frac{1}{8}\times 2\frac{3}{8}$; Maine bricks, $7\frac{1}{2}$ in.$\times 3\frac{3}{4}\times 2\frac{3}{4}$; North River bricks, 8 in.$\times 3\frac{1}{2}\times 2\frac{1}{4}$; and Milwaukee bricks, $8\frac{1}{2}$ in.$\times 4\frac{1}{8}\times 2\frac{3}{8}$.

To build one *square foot* of wall 1 brick or 4 inches thick, requires 7 common bricks; 2 bricks, or 9 in. thick, 14 bricks; 3 bricks, or 13 in. thick, 21 bricks. In practice, the thickness of the wall is regarded as the same for each kind of brick.

Rule I.—*To find the number of perches in a piece of masonry, divide the number of cubic feet by* $24\frac{3}{4}$.

Rule II.—*To find the number of common bricks required for a wall or building, multiply the number of square feet in the wall by* 7, *if the wall is* 1 *brick thick; by* 14, *if* 2 *bricks thick; by* 21, *if* 3 *bricks thick.*

The following is a general rule for all kinds of brick:—***To find the number of any kind of bricks required for a wall, or building, add*** $\frac{1}{4}$ ***of an inch to the length and the thickness of the brick, divide*** 144 ***by the product of these two sums to find the number of bricks in a square foot of wall*** 1 ***brick thick, and multiply by the number of bricks in the thickness, and this product by the number of square feet in the wall.***

NOTE.—An old rule was—*Deduct* $\frac{1}{10}$ *of the solid contents for the mortar, and divide the remainder by the contents of one brick.* We may also *find the contents of a brick with the mortar surrounding it, and divide a cubic foot by this quantity, to find the number of bricks in a cubic foot.*

WRITTEN EXERCISES.

1. How many perches of masonry in a wall 60 ft. long, 4 ft. 6 in. high, and 15 inches thick?

SOLUTION.—Multiplying the length, breadth, and height together, we have $60\times 4\frac{1}{2}\times 1\frac{1}{4}$, or $337\frac{1}{2}$ cu. ft., which, divided by $24\frac{3}{4}$, the number of cubic feet in a perch, equals $13\frac{7}{11}$ perches.

2. What will be the cost of digging a cellar 42 ft. long, 28 ft. wide, and 6 ft. 6 in. deep, at $.42 a load, each load being a cubic yard? *Ans.* $118.90½.

3. How many perches (24¾ cu. ft.) of stone, laid dry, will build a wall around a lot 20 rd. long and 18 rd. wide, the wall to be 5 ft. high and 2 ft. 6 in. thick? *Ans.* 628⅛⅞ perches.

4. What will be the cost of filling in a street 600 feet long and 65 feet wide, averaging 4½ ft. below grade, at $.52 a cubic yard? *Ans.* $3380.

5. How many bricks of average size will it require to build the walls of a house 48 ft. long, 25 ft. wide, and 21 ft. high, the wall being 13 in. thick (21 bricks to sq. ft.), allowing 240 sq. ft. for doors and windows? *Ans.* 57,435 bricks.

SUPPLEMENTARY PROBLEMS.

To be omitted unless otherwise directed.

6. Mr. Wilson had a well dug in his yard, 6 ft. in diameter and 15 ft. 9 in. deep; what did it cost at 50¢ a load? *Ans.* $8.25—.

7. What will be the cost of the bricks in a house 40 ft. square, 22 ft. high, the walls being three bricks thick, of Philadelphia brick, at $15½ per M.? *Ans.* $1027.75.

8. What will be the cost of digging and walling the cellar of a house 45 ft. by 24 ft., the cellar being 6 ft. deep and the wall 7½ ft. high and 1½ ft. thick, if the excavating cost 45¢ a load, and the masonry $4.25 a perch? *Ans.* 374.59\frac{1}{11}$.

MEASURES OF CAPACITY.

365. Measures of Capacity are volumes used to determine the quantity of fluids and many dry substances.

366. The **Principal Measures** of capacity are the *gallon* for liquid substances, and the *bushel* for dry substances.

CAPACITY OF CISTERNS, ETC.

367. The Capacity of Cisterns, etc., is usually expressed in *gallons* or *barrels.*

368. The **Standard Liquid Gallon** of the United States contains 231 cubic inches, and is equal to about 8⅓ lb. Avoirdupois of pure water.

369. The **Barrel** of 31½ gallons, and the hogshead of

63 gallons, are used in measuring the capacity of cisterns, vats, tanks, etc. When used as the names of vessels, these terms express no definite quantity.

The Imperial Gallon of Great Britain contains 277.274 cubic inches, and is equal to about 1.2 U. S. gallons. The beer gallon contains 282 cubic inches, but is now seldom used. A cubic foot of pure water weighs 1000 oz. Avoirdupois.

Rule I.—*To find the capacity of a cistern or vessel in gallons, divide the contents in cubic inches by* 231.

Rule II.—*To find the cubic inches in a given number of gallons, multiply the given number of gallons by* 231.

WRITTEN EXERCISES.

1. How many gallons of water will a tank 6 ft. long, 4 ft. wide, and 2 ft. 3 in. deep, contain?

SOLUTION.—The contents of the tank equal $6 \times 4 \times 2\frac{1}{4}$, which are 54 cubic feet; multiplying by 1728 to reduce to cubic inches, we have 93312 cu. in.; dividing by 231, the number of cubic inches in a gallon, we have $403\frac{7}{11}$ gallons.

2. How many gallons of water are contained in a tank 15 ft. long, 3 ft. wide, and 3 ft. 6 in. deep? *Ans.* $1178\frac{2}{11}$ gallons.

3. How many Imperial gallons would be contained in the same tank? *Ans.* 981.55+ Imp. gal.

4. How many cubic feet in a cistern containing 45 hogsheads? *Ans.* $378\frac{3}{4}$ cu. ft.

5. A cistern 8 ft. square contains 54 hhd.; what is its depth? *Ans.* 7.11— ft.

6. How many barrels of water can be contained in a tank measuring 7 ft. square by 4 ft. deep? *Ans.* $46\frac{6}{11}$ barrels.

7. How many hogsheads of water can be contained in a well whose diameter within the curb is $4\frac{1}{2}$ ft., and depth 12 feet? *Ans.* 22.66+ hhd.

SUPPLEMENTARY PROBLEMS.

To be omitted unless otherwise directed.

8. The diameter of a well is 3.5 ft., and it contains $16\frac{1}{2}$ hhd. of water; what is the depth of the water? *Ans.* 14.44+ ft.

9. A tank 4 yd. long, 2 yd. wide, and 6 ft. deep, is half full of water; what is the weight of the water? *Ans.* 13,500 lb.

10. A tank 7 ft. long, 5 ft. wide, and 3 ft. deep, can be emptied by a waste pipe in 2 hours; how many gallons are discharged in 1 minute? *Ans.* $6\frac{5}{11}$ gallons.

11. Mr. Cornwell constructed a tank in his attic 8 ft. 6 in. long, 4 ft. 3 in. wide, and 2 ft. 6 in. deep; how many hogsheads of water will it hold, and what will be the weight?

Ans. 10.72 + hhd.; $5644\frac{1}{2}$ lb.

12. A reservoir 32 ft. long, 27 ft. wide, and 10 ft. deep, is $\frac{3}{4}$ full when it becomes necessary to draw off the water to clean it out; what will be the expense of pumping the water out at 10 cents a hogshead? *Ans* $76.94+.

CAPACITY OF BINS, ETC.

370. The **Capacity** of **Bins,** etc., is usually expressed in *bushels.*

371. The **Standard Bushel** of the United States is a cylindrical measure $18\frac{1}{2}$ in. in diameter and 8 in. deep, containing 2150.42 cubic inches.

The **Bushel.**—Grain, seeds, and small fruits are sold by *stricken measure.* Potatoes, corn in the ear, large fruits, coal and other bulky articles are sold by *heaped measure.*

In practice we may call 5 *stricken bushels* equal to 4 *heaped bushels.*

Coal.—Coal is bought and sold in large quantities by the *ton;* in small quantities by the *bushel,* 28 heaped bushels, or about 43.5 cu. ft. being considered equal to a ton.

Ordinary *anthracite coal* measures from 36 to 40 cu. ft. to the *ton; bituminous* coal, from 36 to 45 cu. ft. to the ton. Lehigh white ash coal, egg size, measures about $34\frac{1}{2}$ cu. ft. to the ton; Schuylkill white ash 35 cu. ft., and pink, gray, or red ash, 36 cu. ft. to the ton.

Hay.—Hay, when loose or in loads, or upon a scaffold, measures about 500 cu. ft. to the ton; on a mow, 400 cu. ft.; and in large well-settled stacks, 10 cu. yd.

Rule I.—*To find the capacity of a bin in bushels, divide the contents in cubic inches by* 2150.42.

Rule II.—*To find the cubic feet in a given number of bushels, multiply the number of bushels by* 2150.42, *and divide by* 1728.

NOTE.—2150.42 is to 1728 as 5 to 4, nearly; hence a bushel is nearly equa. to $1\frac{1}{4}$ cu. ft. Therefore, for practical purposes, $\frac{4}{5}$ *of the number of cubic feet will equal the number of bushels, and* $\frac{5}{4}$ *of the number of bushels will equal the number of cubic feet.*

Notice also that $\frac{4}{5}$ of the number of bushels *stricken measure* equals the number of bushels *heaped measure.* Examples 5, 9, 10, 11 and 14 are solved by this rule.

WRITTEN EXERCISES.

1. How many bushels will be contained in a bin 8 ft, long, 6 ft. wide, and 3 ft. deep?

SOLUTION.—The contents equal $8\times6\times3$, or 144 cubic feet, which equals 248832 cubic inches; dividing by 2150.42, the number of cubic inches in a bushel, we have 115.71+ bushels.

2. A bin is 16 ft. long, 7 ft. wide, and $2\frac{1}{2}$ ft. deep; how many bushels will it hold? *Ans.* 224.99 bu.

3. What is the width of a bin 24 ft. long and 3 ft. 4 in. deep, to contain 640 bushels of wheat? *Ans.* 9.95+ ft.

4. One division of an elevator is 25 ft. long, 15 ft. wide, and contains 2000 bu. of grain; what is its depth? *Ans.* 6.63+ft.

5. A bin is 10 ft. 5 in. long, 7 ft. wide, and 3 ft. 6 in. deep; how many bushels of shelled corn will it hold, if $\frac{4}{5}$ of the number of cu. ft. equals the number of bushels? *Ans.* $204\frac{1}{2}$ bu.

6. A bin 5 ft. long, $4\frac{1}{2}$ ft. wide, and 3 ft. deep, is filled with Schuylkill white ash coal (35 cu. ft.); what is its value at \$6.50 a ton? *Ans.* \$12.53$\frac{1}{2}$.

7. A shed 6 yd. long, $4\frac{1}{2}$ yd. wide, and 8ft. high, is half full of Lehigh white ash coal ($34\frac{1}{2}$ cu. ft.); what is the value of the coal at \$7.50 a ton? *Ans.* \$211.304.

8. An ice-house 40 ft. long, 25 ft. wide, and 15 ft. high, is filled with ice; how many tons are there, if a cubic foot weighs $58\frac{1}{3}$ lb.? *Ans.* $435\frac{5}{14}$ tons.

SUPPLEMENTARY PROBLEMS.

To be omitted unless otherwise directed.

9. There is a rectangular box 4 ft. long, 3 ft. wide, and 2 ft. 4 in. deep; how many bushels of apples will it hold? how many of cranberries? *Ans.* $17\frac{2}{5}$ bu.; $22\frac{2}{5}$ bu.

10. A bin 8 ft. long, 6 ft. wide, and 3 ft deep, is $\frac{3}{4}$ full of barley; what is its value at \$1.25 a bushel? *Ans.* \$108.

11. How many barrels of flour (196 lbs.) can be made from the contents of a bin 12 ft. long, $7\frac{1}{2}$ ft. wide, and 4 ft. deep, if one bushel of wheat makes 48 lb. of flour? *Ans.* $70\frac{4}{7}$ lb.

12. A hay-mow is 24 ft. long by 18 ft. wide and 16 ft. high; what is the value of the hay when it is filled, valued at $12 a ton, 400 cu. ft. to a ton? *Ans.* $207.36.

13. Mr. Jenkins sold a rectangular stack of hay 8 ft. long, 7 ft. wide, and 6 ft. high, at $15 a ton; what was the value of the hay, 10 cu. yd. to a ton? *Ans.* $18.66⅔.

14. A crib filled with corn in the ear measures on the inside 20 ft. in length, 12 ft. in width, and 7 ft. in height; what will be the value of the corn when shelled, at $1.05 a bushel, if 2 bushels in the ear make 1 bushel when shelled? *Ans.* $564.48.

MISCELLANEOUS PROBLEMS.

IN PRACTICAL MEASUREMENTS.

1. How many planks laid crosswise, 1 ft. wide, will it take for a board walk 1 mi. 16 rd. long, and 4 ft. wide? *Ans.* 5544.

2. How many bushels of grain can a farmer store in a hogshead containing 122 gallons? *Ans.* 13.1+bu.

3. How much stair carpet will be required for a flight of 18 steps, each 10 in. wide and 7½ in. high? *Ans.* 8¾ yd.

4. To dig a sewer 1½ miles long, 5 ft. wide, and 8 ft. deep, cost $3716; what was the price per cubic yard? *Ans.* $.31⅛⅞.

5. A has a mow 20 ft. long, 12 ft. wide, and 10 ft. deep; how many tons of hay (400 cu. ft.) does it hold? *Ans.* 6 tons.

6. From a quartz rock yielding silver at the rate of $123.75 per ton, a miner obtained $75.64 worth; what was the weight of the rock? *Ans.* 12 cwt. 22⅛⅝ lb.

7. If 4 persons can stand on one square yard of ground, how many people can be contained in a public square 32 rods on each side? *Ans.* 123904.

8. How many bunches of lath will be required for the walls and ceiling of a room 18 ft. long, 14 ft. wide, 10 ft. high, each bunch being estimated to cover 5 sq. yd.? *Ans.* 19¾⅓.

9. I wish to cover my parlor 25 ft.×17 ft. 6 in. with carpet 26 in. wide; what will it cost me at $1.87½ per yard, the pattern requiring an allowance of 2⅜ yd. for waste? *Ans.* $131.25.

10. A street 36 ft. wide was paved with Nicholson pavement, at $3.25 per square yard; what did it cost to pave a "square" 32 rods long? *Ans.* $6864.

11. A railroad tunnel is one-eighth of a mile long, averaging 24 ft. wide and 20 ft. high; what did the excavation cost, at $1.50 a cubic yard? *Ans.* $17600.

12. How many freight cars will be required to transport 12000 bu. of wheat, 60 lb. each, 24000 lb. being the weight allowed for a single car? *Ans.* 30 cars.

13. A coal dealer has a wagon which holds exactly one ton of Schuylkill red ash coal (36 cu. ft.); if the wagon-bed is 7½ ft. long and 4½ ft. wide, what is its depth? *Ans.* 12½ in.

14. I used the earth taken from 4 cellars in grading a lot of ground; if the cellars were 30×21 ft., 28×18 ft., 24×16 ft., and 32×24 ft., respectively, and 5 ft. deep, how many loads (cu. yd.) did I use? *Ans.* 423⅓ loads.

SUPPLEMENTARY PROBLEMS.

To be omitted unless otherwise directed.

15. What costs the excavation for a cellar 5½ ft. deep under the main building of a dwelling-house 30×25 ft. and an excavation for the walls of an L 16 ft. square, 1½ ft. wide, and 2 ft. deep, at 50¢ per cu. yd.? *Ans.* $78.88⅛.

16. What costs the plastering of a house of 12 rooms, there being on each story 4 rooms 14×15 ft. and a hall 30×8 ft.; the first story being 10 ft. high, the second 9½ ft., and the third 8½ ft., allowance being also made for 24 doors 7×3½ ft., and 30 windows 6×3 ft., at 30¢ per sq. yd.? *Ans.* $357.86⅔.

17. Required the cost of a cellar of a house 40×30 ft., the different items being as follows: excavating cellar, 4 ft. deep, at 50¢ per cu. yd.; cellar wall, 7 ft. high and 18 in. thick, the lower 4 ft. common masonry, @$3.15 a perch, and the upper 3 ft. cut stone at 16¢ per sq. ft. *Ans.* $263—.

18. Required the cost of brick-work of the same house, the walls being 35 ft. high and 13 in. thick, and gable 10 ft. high, using common bricks at $10 per M., bricklaying costing $3 per M., allowing for 3 doors, each 7 ft. by 3½ ft., and 30 windows, each 6 ft. by 3 ft. *Ans.* $1258.915.

SUPPLEMENT TO DENOMINATE NUMBERS.

These supplementary exercises of Arts, 372, 373 and 374 are to be omitted by pupils needing a shorter course, unless otherwise directed.

COMPARISON OF MEASURES OF CAPACITY.

372. The **Dry Gallon,** or *half peck,* contains 268.8 cubic inches; hence 6 dry gallons equal nearly 7 liquid gallons.

NOTE.—The pupil will remember that the liquid gallon contains 231 cu. in., and the old beer gallon 282 cu. in.

WRITTEN EXERCISES.

1. How many more cubic inches in 495.3 dry gallons than in 495.3 liquid gallons? *Ans.* 18722.34 cu. in.

2. What part of 5 gal. 2 qt. of old beer measure is 3 gal. 2 qt. 1 pt. of liquid measure? *Ans.* .5399—.

3. How many bushels of oats can be put in a tank that holds 4550 gallons of water? *Ans.* 488.765— bu.

4. How many gallons of water can be poured into a bushel measure to exactly fill it? *Ans.* 9.31—.

5. If 32 quarts of water are put into a vessel that holds exactly 32 quarts of strawberries, will the water be more or less than the vessel will hold? *Ans.* 302.4 cu. in. less.

6. A grocer sold molasses at 15 cents a quart, but the clerk, by mistake, measured a day's sales by the dry quart; he sold 25 gallons; what did it cost a quart, liquid measure?

Ans. $12\frac{7}{8}\frac{7}{4}$ cents.

COMPARISON OF WEIGHTS.

373. The **Troy Pound** and the **Apothecaries' Pound** each contains 5760 Troy grains; the *Avoirdupois* pound contains 7000 Troy grains. The Avoirdupois ounce contains 437½ grains, while the Troy ounce contains 480 grains.

Rule.—*Reduce the given denominations to grains, and divide by* 480 *to find Troy ounces, and* 437½ *to find ounces Avoirdupois.*

1. How many rings, each weighing 2⅞ pwt., can be made from a bar of gold weighing 1 lb. Avoirdupois?

Ans. 101 rings, 31 gr. remaining.

2. What is the value of a silver pitcher weighing 2lb. 10 oz. Avoirdupois, at $2.25 per ounce Troy? *Ans.* $86.13.

3. Which is the heavier, 52⅝ lb. of lead or 52.625 lb. of silver? *Ans.* The lead.

4. How many pounds of gold are actually as heavy as 10 lb. of iron? *Ans.* 12½½ lb.

5. How many times 3 lb. 10 oz. Troy is 5lb. 12 oz. Avoirdupois? *Ans.* 1.8229+.

6. A young lady weighs 120 lb. Avoirdupois weight; how much would she weigh by Troy weight? *Ans.* 145⅗ lb.

7. If a druggist buys 25 lb. Avoirdupois of drugs at $8½ a pound, and sells them in prescriptions at the rate of 75¢ an ℥, what is the gain? *Ans.* $65.10.

COMPARISON OF MONEY.

374. The **Pound Sterling** is valued at $4.8665; and the *Franc* at 19.3 cents; the *Mark* at 23.85 cents.

375. The gold **Dollar** weighs 25.8 gr.; the silver *Dollar*, 412½ gr.; the *Half-dollar*, 192.9 gr.; the *Sovereign* (gold), 123.274 gr.; and the *Shilling* (silver), 87.27 gr.

1. How many dollars in £25? *Ans.* $121.66¼.

2. How many dollars in £14 12s? *Ans.* $71.05.

3. How many pounds in $256.16? *Ans.* £52 12s. 9d.—.

4. How many dollars in 240 guineas? *Ans.* $1226.358.

5. How many francs in £42? *Ans.* 1059.03 fr.

6. How many pounds in 875 francs? *Ans.* £34 14.03 s.

7. How many marks in £340? *Ans.* 6937.568 marks.

8. How many pounds in 4375 marks? *Ans.* £214 8s. 3d.—.

9. How many pounds Avoirdupois in 1000 sovereigns? *Ans.* 17.610¼.

10. An ingot of pure gold was brought from California to be coined at the Philadelphia mint; if it weighed I5 lb. 8 oz. Troy, how many gold dollars will it make, $\frac{1}{10}$ of the coin being alloy? *Ans.* $3886; 7.08 gr. rem.

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